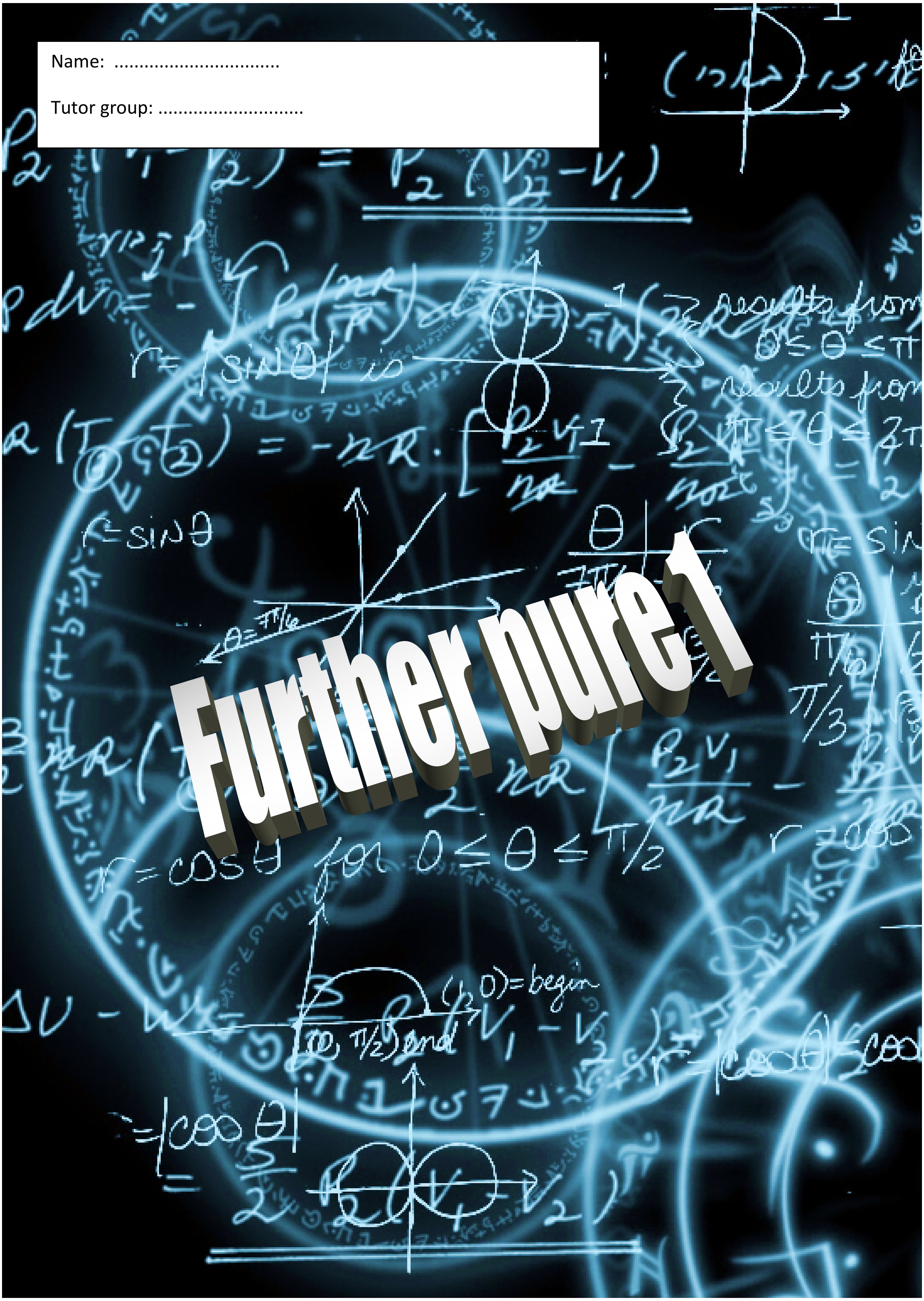
AQA Further pure maths 1

Name: ....

Tutor group: ....



Key dates

Further pure 1 exam : **17th May 2013 am**

Term dates:

Term 1: Thursday 1 September 2011 - Friday 21 October 2011 (37 teaching days)	**Term 4: Monday 20 February 2012 - Friday 30 March 2012 (30 teaching days)**
Term 2: Monday 31 October 2011 - Friday 16 December 2011 (35 teaching days)	**Term 5: Monday 16 April 2012 - Friday 1 June 2012 (34 teaching days)**
Term 3: Tuesday 3 January 2012- Friday 10 February 2012 (29 teaching days)	**Term 6: Monday 11 June 2012 - Thursday 19 July 2012 (29 teaching days)**

Scheme of Assessment *Further Mathematics Advanced Subsidiary (AS) Advanced Level (AS + A2)*

Candidates for AS and/or A Level Further Mathematics are expected to have already obtained (or to be obtaining concurrently) an AS and/or A Level award in Mathematics.

The Advanced Subsidiary (AS) award comprises three units chosen from the full suite of units in this specification, except that the Core
units cannot be included. One unit must be chosen from MFP1, MFP2, MFP3 and MFP4. All three units can be at AS standard; for example, MFP1, MM1B and MS1A could be chosen. All three units can be in Pure Mathematics; for example, MFP1, MFP2 and MFP4 could be chosen.

The Advanced (A Level) award comprises six units chosen from the full suite of units in this specification, except that the Core units cannot be included. The six units must include at least two units from MFP1, MFP2, MFP3 and MFP4. All four of these units could be chosen. At least three of the six units counted towards A Level Further Mathematics must be at A2 standard.

All the units count for $33\frac{1}{3}$% of the total AS marks
$16\frac{2}{3}$% of the total A level marks

Written Paper
1hour 30 minutes
75 marks

Further Pure 1

All questions are compulsory. A graphics calculator may be used

Grading System

The AS qualifications will be graded on a five-point scale: A, B, C, D and E. The full A level qualifications will be graded on a six-point scale: A*, A, B, C, D and E.
To be awarded an A* in Further Mathematics, candidates will need to achieve grade A on the full A level qualification and 90% of the maximum uniform mark on the aggregate of the best three of the A2 units which contributed towards Further Mathematics. For all qualifications, candidates who fail to reach the minimum standard for grade E will be recorded as U (unclassified) and will not receive a qualification certificate.

Title	Type	Unit	Award	Name	Max	Grade (UMS/Points) Boundaries								
Mathematics	GCE	MD01		Decision 1	100 UMS		A(80)	B(70)	C(60)	D(50)	E(40)			
		MD02		Decision 2	100 UMS		A(80)	B(70)	C(60)	D(50)	E(40)			
		MFP1		Further Pure Mathematics 1	100 UMS		A(80)	B(70)	C(60)	D(50)	E(40)			

Further pure 1 subject content

Algebra and Graphs
Complex Numbers
Roots and Coefficients of a quadratic equation
Series
Calculus
Numerical Methods
Trigonometry
Matrices and Transformations

Further pure 1 specifications

Candidates will be expected to be familiar with the knowledge, skills and understanding implicit in the modules Core 1 and Core 2.

Candidates will also be expected to know for section 6 that the roots of an equation f(x)=0 can be located by considering changes of sign of f(x) in an interval of x in which f(x) is continuous.

Candidates may use relevant formulae included in the formulae booklet without proof.

<table>
<tr><th colspan="2">ALGEBRA AND GRAPHS</th></tr>
<tr><td>Graphs of rational functions of the form
$\frac{ax+b}{cx+d}$, $\frac{ax+b}{cx^2+dx+e}$ or
$\frac{x^2+ax+b}{x^2+cx+d}$</td><td>Sketching the graphs.

Finding the equations of the asymptotes which will always be parallel to the coordinate axes.
Finding points of intersection with the coordinate axes or other straight lines.
Solving associated inequalities.
Using quadratic theory (not calculus) to find the possible values of the function and the coordinates of the maximum or minimum points on the graph.
E.g.
$for\ y=\frac{x^2+2}{x^2-4x}, y=k \Rightarrow x^2+2=kx^2-4kx,$
$which\ has\ real\ roots\ if\ 16k^2+8k-8\geq 0,$
$i.e.\ if\ k\leq -1\ or\ k\geq -\frac{1}{2};$
$\text{Stationary points are } (1,-1) \text{ and } (-2,\frac{1}{2})$</td></tr>
<tr><td>Graphs of parabolas, ellipses and hyperbolas with equations
$y^2=4ax, \frac{x^2}{a^2}+\frac{y^2}{b^2}=1,$
$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1\ and\ xy=c^2$</td><td>Sketching the graphs.

Finding points of intersection with the coordinate axes or other straight lines. Candidates will be expected to interpret the geometrical implication of equal roots, distinct real roots or no real roots.
Knowledge of the effects on these equations of single transformations of these graphs involving translations, stretches parallel to the x- or y-axes, and reflections in the line $y = x$.
Including the use of the equations of the asymptotes of the hyperbolas given in the formulae booklet.</td></tr>
<tr><td colspan="2"></td></tr>
<tr><th colspan="2">COMPLEX NUMBERS</th></tr>
<tr><td>Non-real roots of quadratic equations.</td><td>Complex conjugates - awareness that non-real roots of quadratic equations with real coefficients occur in conjugate pairs.</td></tr>
<tr><td>Sum, difference and product
of complex numbers in the form x+iy.</td><td></td></tr>
<tr><td>Comparing real and imaginary parts.</td><td>Including solving equations
E.g. $Solving\ 2z+z^*=1+i\ \ where\ \ z^*\ is\ the\ conjugate\ of\ z$</td></tr>
</table>

ROOTS AND COEFFICENTS OF A QUADRATIC EQUATION

Manipulating expressions involving $\alpha+\beta$ and $\alpha\beta$	E.g. $\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$ Forming an equation with roots α^3,β^3 *or* $\frac{1}{\alpha},\frac{1}{\beta}$ *or* $\alpha+\frac{2}{\beta},\beta+\frac{2}{\alpha}$ *etc.*

SERIES

Use of formulae for the sum of the squares and the sum of the cubes of the natural numbers.	E.g. Find a polynomial expression for $\sum_{r=1}^{n} r^2(r+1)$ *or* $\sum_{r=1}^{n}(r^2-r+1)$

CALCULUS

Finding the gradient of the tangent to a curve at a point, by taking the limit as *h* tends to zero of the gradient of a chord joining two points whose *x*-coordinates differ by *h*	The equation will be given as $y=f(x)$, E.g. where $f(x)$ is a simple polynomial such as x^2-2x *or* x^4+3
Evaluation of simple improper integrals.	E.g. $\int_0^4 \frac{1}{\sqrt{x}}dx$, $\int_4^\infty x^{-\frac{3}{2}}dx$

NUMERICAL METHODS

Finding roots of equations by interval bisection, linear interpolation and the Newton-Raphson method.	Graphical illustration of these methods.
Solving differential equations of the form $\frac{dy}{dx}=f(x)$	Using a step-by-step method based on the linear approximations $y_{n+1}\approx y_n+hf(x_n)$; $x_{n+1}=x_n+h$, *with given values for* x_0, y_0 *and* h
Reducing a relation to a linear law.	E.g. $\frac{1}{x}+\frac{1}{y}=k$; $y^2=ax^3+b$; $y=ax^n$; $y=ab^x$ Use of logarithms to base 10 where appropriate. Given numerical values of *(x,y)*, drawing a linear graph and using it to estimate the values of the unknown constants.

TRIGONOMETRY

General solutions of trigonometric equations including use of exact values for the sine, cosine and tangent of $\frac{\pi}{6},\frac{\pi}{4},\frac{\pi}{3}$	E.g. $Sin2x=\frac{\sqrt{3}}{2}$, $Cos\left(x+\frac{\pi}{6}\right)=-\frac{1}{\sqrt{2}}$, $Tan\left(\frac{\pi}{3}-2x\right)=1$ $Sin2x=0.3$, $Cos(3x-1)=-0.2$

MATRICES

2x2 and 2x1 matrices; addition and subtraction, multiplication by a scalar. Multiplying a 2x2 matrix by a 2x2 matrix or by a 2x1 matrix. The identity matrix **I** for a 2x2 matrix.	Transformations will be restricted to rotations about the origin, reflections in a line through the origin, stretches parallel to the x- and y- axes, and enlargements with centre the origin. Use of the standard transformation matrices given in the formulae booklet.
Transformations of points in the *x-y* plane represented by 2x2 matrices.	Combinations of these transformations

The formulae booklet

Mensuration

Surface area of sphere $= 4\pi r^2$

Area of curved surface of cone $= \pi r \times$ slant height

Arithmetic series

$$u_n = a + (n-1)d$$

$$S_n = \tfrac{1}{2}n(a+l) = \tfrac{1}{2}n\left[2a+(n-1)d\right]$$

Geometric series

$$u_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Summations

$$\sum_{r=1}^{n} r = \tfrac{1}{2}n(n+1)$$

$$\sum_{r=1}^{n} r^2 = \tfrac{1}{6}n(n+1)(2n+1)$$

$$\sum_{r=1}^{n} r^3 = \tfrac{1}{4}n^2(n+1)^2$$

Trigonometry – the Cosine rule

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Binomial Series

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$$

$$\text{where } \binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1.2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1.2\ldots r}x^r + \ldots \qquad (|x| < 1, n \in \mathbb{R})$$

Logarithms and exponentials

$$a^x = e^{x\ln a}$$

Complex numbers

$$\{r(\cos\theta + i\sin\theta)\}^n = r^n(\cos n\theta + i\sin n\theta)$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

The roots of $z^n = 1$ are given by $z = e^{\frac{2\pi k i}{n}}$, for $k = 0, 1, 2, \ldots, n-1$

Maclaurin's series

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \ldots + \frac{x^r}{r!} f^{(r)}(0) + \ldots$$

$$e^x = \exp(x) = 1 + x + \frac{x^2}{2!} + \ldots + \frac{x^r}{r!} + \ldots \quad \text{for all } x$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots + (-1)^{r+1}\frac{x^r}{r} + \ldots \quad (-1 < x \leqslant 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + (-1)^r \frac{x^{2r+1}}{(2r+1)!} + \ldots \quad \text{for all } x$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots + (-1)^r \frac{x^{2r}}{(2r)!} + \ldots \quad \text{for all } x$$

Hyperbolic functions

$$\cosh^2 x - \sinh^2 x = 1$$

$$\sinh 2x = 2\sinh x \cosh x$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

$$\cosh^{-1} x = \ln\left\{x + \sqrt{x^2 - 1}\right\} \quad (x \geqslant 1)$$

$$\sinh^{-1} x = \ln\left\{x + \sqrt{x^2 + 1}\right\}$$

$$\tanh^{-1} x = \tfrac{1}{2}\ln\left(\frac{1+x}{1-x}\right) \quad (|x| < 1)$$

Conics

	Ellipse	Parabola	Hyperbola	Rectangular hyperbola
Standard form	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$y^2 = 4ax$	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$xy = c^2$
Asymptotes	none	none	$\frac{x}{a} = \pm\frac{y}{b}$	$x = 0, y = 0$

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad \left(A \pm B \neq (k + \tfrac{1}{2})\pi\right)$$

$$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$$

$$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$$

Vectors

The resolved part of **a** in the direction of **b** is $\dfrac{\mathbf{a}.\mathbf{b}}{|\mathbf{b}|}$

The position vector of the point dividing AB in the ratio $\lambda:\mu$ is $\dfrac{\mu\mathbf{a}+\lambda\mathbf{b}}{\lambda+\mu}$

Vector product: $\mathbf{a}\times\mathbf{b}=|\mathbf{a}||\mathbf{b}|\sin\theta\,\hat{\mathbf{n}}=\begin{vmatrix}\mathbf{i} & a_1 & b_1\\ \mathbf{j} & a_2 & b_2\\ \mathbf{k} & a_3 & b_3\end{vmatrix}=\begin{bmatrix}a_2b_3-a_3b_2\\ a_3b_1-a_1b_3\\ a_1b_2-a_2b_1\end{bmatrix}$

If A is the point with position vector $\mathbf{a}=a_1\mathbf{i}+a_2\mathbf{j}+a_3\mathbf{k}$ and the direction vector **b** is given by $\mathbf{b}=b_1\mathbf{i}+b_2\mathbf{j}+b_3\mathbf{k}$, then the straight line through A with direction vector **b** has cartesian equation

$$\frac{x-a_1}{b_1}=\frac{y-a_2}{b_2}=\frac{z-a_3}{b_3}=\lambda$$

The plane through A with normal vector $\mathbf{n}=n_1\mathbf{i}+n_2\mathbf{j}+n_3\mathbf{k}$ has cartesian equation

$n_1x+n_2y+n_3z=d$ where $d=\mathbf{a}.\mathbf{n}$

The plane through non-collinear points A, B and C has vector equation

$\mathbf{r}=\mathbf{a}+\lambda(\mathbf{b}-\mathbf{a})+\mu(\mathbf{c}-\mathbf{a})=(1-\lambda-\mu)\mathbf{a}+\lambda\mathbf{b}+\mu\mathbf{c}$

The plane through the point with position vector **a** and parallel to **b** and **c** has equation

$\mathbf{r}=\mathbf{a}+s\mathbf{b}+t\mathbf{c}$

Matrix transformations

Anticlockwise rotation through θ about O: $\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$

Reflection in the line $y=(\tan\theta)x$: $\begin{bmatrix}\cos 2\theta & \sin 2\theta\\ \sin 2\theta & -\cos 2\theta\end{bmatrix}$

The matrices for rotations (in three dimensions) through an angle θ about one of the axes are

$\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\theta & -\sin\theta\\ 0 & \sin\theta & \cos\theta\end{bmatrix}$ for the x-axis

$\begin{bmatrix}\cos\theta & 0 & \sin\theta\\ 0 & 1 & 0\\ -\sin\theta & 0 & \cos\theta\end{bmatrix}$ for the y-axis

$\begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$ for the z-axis

Differentiation

$\mathbf{f}(x)$	$\mathbf{f}'(x)$
$\sin^{-1} x$	$\dfrac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$-\dfrac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\dfrac{1}{1+x^2}$
$\tan kx$	$k\sec^2 kx$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\operatorname{sech}^2 x$
$\sinh^{-1} x$	$\dfrac{1}{\sqrt{1+x^2}}$
$\cosh^{-1} x$	$\dfrac{1}{\sqrt{x^2-1}}$
$\tanh^{-1} x$	$\dfrac{1}{1-x^2}$
$\dfrac{\mathrm{f}(x)}{\mathrm{g}(x)}$	$\dfrac{\mathrm{f}'(x)\,\mathrm{g}(x)-\mathrm{f}(x)\,\mathrm{g}'(x)}{(\mathrm{g}(x))^2}$

Integration

(+ constant; $a > 0$ where relevant)

$\mathbf{f}(x)$	$\int \mathbf{f}(x)\,\mathbf{d}x$
$\tan x$	$\ln\lvert\sec x\rvert$
$\cot x$	$\ln\lvert\sin x\rvert$
$\operatorname{cosec} x$	$-\ln\lvert\operatorname{cosec} x+\cot x\rvert = \ln\lvert\tan(\tfrac{1}{2}x)\rvert$
$\sec x$	$\ln\lvert\sec x+\tan x\rvert = \ln\lvert\tan(\tfrac{1}{2}x+\tfrac{1}{4}\pi)\rvert$
$\sec^2 kx$	$\dfrac{1}{k}\tan kx$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\ln\cosh x$

$\dfrac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\left(\dfrac{x}{a}\right) \quad (\lvert x\rvert < a)$
$\dfrac{1}{a^2+x^2}$	$\dfrac{1}{a}\tan^{-1}\left(\dfrac{x}{a}\right)$
$\dfrac{1}{\sqrt{x^2-a^2}}$	$\cosh^{-1}\left(\dfrac{x}{a}\right)$ or $\ln\{x+\sqrt{x^2-a^2}\} \quad (x > a)$
$\dfrac{1}{\sqrt{a^2+x^2}}$	$\sinh^{-1}\left(\dfrac{x}{a}\right)$ or $\ln\{x+\sqrt{x^2+a^2}\}$
$\dfrac{1}{a^2-x^2}$	$\dfrac{1}{2a}\ln\left\lvert\dfrac{a+x}{a-x}\right\rvert = \dfrac{1}{a}\tanh^{-1}\left(\dfrac{x}{a}\right) \quad (\lvert x\rvert < a)$
$\dfrac{1}{x^2-a^2}$	$\dfrac{1}{2a}\ln\left\lvert\dfrac{x-a}{x+a}\right\rvert$

$$\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$$

a of a sector

$$A = \frac{1}{2}\int r^2\,\mathrm{d}\theta \quad \text{(polar coordinates)}$$

length

$$s = \int\sqrt{1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\,\mathrm{d}x \quad \text{(cartesian coordinates)}$$

$$s = \int\sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2+\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2}\,\mathrm{d}t \quad \text{(parametric form)}$$

face area of revolution

$$S_x = 2\pi\int y\sqrt{1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\,\mathrm{d}x \quad \text{(cartesian coordinates)}$$

$$S_x = 2\pi\int y\sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2+\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2}\,\mathrm{d}t \quad \text{(parametric form)}$$

nerical integration

The trapezium rule: $\int_a^b y\,\mathrm{d}x \approx \tfrac{1}{2}h\{(y_0+y_n)+2(y_1+y_2+\ldots+y_{n-1})\}$, where $h=\dfrac{b-a}{n}$

The mid-ordinate rule: $\int_a^b y\,\mathrm{d}x \approx h(y_{\frac{1}{2}}+y_{\frac{3}{2}}+\ldots+y_{n-\frac{3}{2}}+y_{n-\frac{1}{2}})$, where $h=\dfrac{b-a}{n}$

Simpson's rule: $\int_a^b y\,\mathrm{d}x \approx \tfrac{1}{3}h\{(y_0+y_n)+4(y_1+y_3+\ldots+y_{n-1})+2(y_2+y_4+\ldots+y_{n-2})\}$

where $h=\dfrac{b-a}{n}$ and n is even

Numerical solution of differential equations

For $\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x)$ and small h, recurrence relations are:

Euler's method: $y_{n+1} = y_n + h\,\mathrm{f}(x_n)$; $\quad x_{n+1} = x_n + h$

For $\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x, y)$:

Euler's method: $y_{r+1} = y_r + h\,\mathrm{f}(x_r, y_r)$

Improved Euler method: $y_{r+1} = y_r + \frac{1}{2}(k_1 + k_2)$, where $k_1 = h\,\mathrm{f}(x_r, y_r)$, $k_2 = h\,\mathrm{f}(x_r + h, y_r + k_1)$

Numerical solution of equations

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \frac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Content

ALGEBRA AND GRAPHS

Key point summary

1 An asymptote is a line that a curve approaches for large values of $|x|$ or $|y|$. It is usually represented by a broken or dotted line.

2 The line $x = a$ is a vertical asymptote of the curve $y = \dfrac{f(x)}{g(x)}$ if $g(a) = 0$.

3 To find a horizontal asymptote of $y = \dfrac{ax+b}{cx+d}$, re-write the equation as $y = \dfrac{a + \dfrac{b}{x}}{c + \dfrac{d}{x}}$.

As $|x| \to \infty$, $\dfrac{1}{x} \to 0$, therefore $y \to \dfrac{a}{c}$.

The horizontal asymptote has equation $y = \dfrac{a}{c}$.

4 In order to solve inequalities such as

$$\frac{ax+b}{cx+d} < k \quad \text{or} \quad \frac{ax+b}{cx+d} > k:$$

1. Sketch the graph of $y = \dfrac{ax+b}{cx+d}$ and the line $y = k$.
2. Solve the equation $\dfrac{ax+b}{cx+d} = k$.
3. Use the graph to find the possible values of x for which the graph lies below or above the line.

Key point summary

1 It is always useful to draw any asymptotes as the first stage in sketching the graph of a rational function.

2 When a curve has a vertical asymptote at $x = a$, it is useful to check the values of y when x is a little smaller than a and when x is a little larger than a.

By considering the behaviour very close to the asymptotes, it is often possible to deduce the main shape of the graph.

3 In order to find the set of values of y for which a curve of the form $y = \dfrac{ax^2 + bx + c}{dx^2 + ex + f}$ exists:

1. consider where $y = k$ cuts the curve by writing $k = \dfrac{ax^2 + bx + c}{dx^2 + ex + f}$;
2. multiply out to obtain a quadratic of the form $Ax^2 + Bx + C = 0$, where A, B and C will involve k;
3. use the condition for real roots $B^2 - 4AC \geqslant 0$ to obtain a quadratic inequality involving k,
4. convert the solution involving k to a condition involving y.

4 If a curve of the form $y = \dfrac{ax^2 + bx + c}{dx^2 + ex + f}$ is shown to exist only for $P \leqslant y \leqslant Q$, then it means that the curve must have a minimum point when $y = P$ and a maximum point when $y = Q$.

Substitute $y = P$ in order to find the x-coordinate of the minimum point. The resulting quadratic in x will always have a repeated root.

Repeat by substituting $y = Q$ to find the x-coordinate of the maximum point.

5 If a curve of the form $y = \dfrac{ax^2 + bx + c}{dx^2 + ex + f}$ is shown to exist only for $y \leqslant M$ or $y \geqslant N$, then it means that the curve must have a maximum point when $y = M$ and a minimum point when $y = N$.

Substitute $y = M$ in order to find the x-coordinate of the maximum point. The resulting quadratic in x will always have a repeated root.

Repeat by substituting $y = N$ to find the x-coordinate of the minimum point.

Conics

Key point summary

1 A reflection in the line $y = x$ maps (x, y) onto (x', y'), where $\begin{matrix} x' = y \\ y' = x \end{matrix}$.

The equation of the new curve after a reflection in the line $y = x$ is obtained by interchanging x and y in the original equation.

2 A parabola with its vertex at the origin and its axis along the x-axis will have an equation of the form $y^2 = kx$, where k is a constant.

3 A stretch of scale factor c in the x-direction and scale factor d in the y-direction, maps (x, y) onto (x', y'), where $\begin{matrix} x' = cx \\ y' = dy \end{matrix}$.

The equation of the new curve is obtained by replacing x by $\left(\frac{x}{c}\right)$ and y by $\left(\frac{y}{d}\right)$ in the original equation.

4 The general equation of an ellipse with its centre at the origin is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

It cuts the x-axis when $x = \pm a$ and cuts the y-axis when $y = \pm b$.

5 An equation of the form $xy = c^2$ represents a hyperbola with the coordinate axes as asymptotes. It is often referred to as a rectangular hyperbola.

It can be rotated through 45° to give an equation of the form $x^2 - y^2 = k$.

6 The equation $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ represents a hyperbola with centre at the origin cutting the x-axis when $x = \pm a$.

It does not intersect the y-axis and its asymptotes have equations $y = \pm\frac{b}{a}x$. When $b = a$ the hyperbola is said to be a rectangular hyperbola.

7 A translation with vector $\begin{bmatrix} c \\ d \end{bmatrix}$ maps (x, y) onto (x', y'),

where $\begin{matrix} x' = x + c \\ y' = y + d \end{matrix}$.

The equation of the new curve is obtained by replacing x by $(x - c)$ and y by $(y - d)$ in the original equation.

8 To consider how a straight line intersects a parabola, ellipse or hyperbola:

1. form a quadratic equation (usually in terms of x) of the form $ax^2 + bx + c = 0$;
2. when $b^2 - 4ac < 0$ there are no points of intersection;
3. when $b^2 - 4ac > 0$ there are two distinct points of intersection;
4. when $b^2 - 4ac = 0$ there is a single point of intersection. The line is a tangent to the curve at that point.

COMPLEX NUMBERS

Key point summary

1 The square root of -1 is not real. Because it is imaginary, it is denoted by i where $i^2 = -1$.

2 Since $i^2 = -1$, it follows that $i^4 = 1$.
The powers of i form a periodic cycle of the form $i, -1, -i, +1, \ldots$, so that $i^5 = i$, $i^6 = -1$, etc.

3 The equation $x^2 = -1$ has the two solutions:
$x = i$ and $x = -i$.

4 A number of the form $p + qi$, where p and q are real numbers and $i^2 = -1$, is called a complex number.

5 The complex conjugate of $p + qi$ is $p - qi$, where p and q are real numbers.
The conjugate of z is denoted by z^*.

6 When a quadratic equation with real coefficients has complex roots, these roots are always a pair of complex conjugates.

7 In general, when $z = p + qi$, where p and q are real numbers,
- the real part of z is p, and
- the imaginary part of z is q.

Roots and Coefficients of a quadratic equation

Key point summary

1 When the quadratic equation $ax^2 + bx + c = 0$ has roots α and β:

- The sum of the roots, $\boxed{\alpha + \beta = -\frac{b}{a};}$
- and the product of roots, $\boxed{\alpha\beta = \frac{c}{a}.}$

2 A quadratic equation can be expressed as $\boxed{x^2 - (\text{sum of roots})x + (\text{product of roots}) = 0}$

3 Two useful results are:

$$\boxed{\begin{aligned} \alpha^2 + \beta^2 &= (\alpha + \beta)^2 - 2\alpha\beta \\ \alpha^3 + \beta^3 &= (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta). \end{aligned}}$$

4 The basic method for forming new equations with roots that are related to the roots of a given equation is:

1. Write down the sum of the roots, $\alpha + \beta$, and the product of the roots, $\alpha\beta$, of the given equation.
2. Find the sum and product of the new roots in terms of $\alpha + \beta$ and $\alpha\beta$.
3. Write down the new equation using

$$x^2 - (\text{sum of new roots})x + (\text{product of new roots}) = 0$$

Series

Key point summary

1 $\displaystyle\sum_{r=1}^{n} 1 = 1 + 1 + 1 + \ldots + 1 = n$

2 $\displaystyle 1 + 2 + 3 + 4 + \ldots + n = \sum_{r=1}^{n} r = \tfrac{1}{2}n(n + 1)$

3 $\displaystyle \sum\left[Af(r) + Bg(r)\right] = A\sum f(r) + B\sum g(r)$

4 $\displaystyle 1^2 + 2^2 + 3^2 + 4^2 + \ldots + n^2 = \sum_{r=1}^{n} r^2 = \tfrac{1}{6}n(n + 1)(2n + 1)$

5 $\displaystyle 1^3 + 2^3 + 3^3 + 4^3 + \ldots + n^3 = \sum_{r=1}^{n} r^3 = \tfrac{1}{4}n^2(n + 1)^2$

Calculus

Key point summary

1 The gradient of the chord PQ is $\dfrac{y_Q - y_P}{x_Q - x_P}$.

2 When the x-coordinate of P is a, and the x-coordinate of Q is $a + h$, the gradient of the chord PQ can be simplified to an expression involving h.
The gradient of the curve at the point P is obtained by letting h tend to zero.

3 When $f(x)$ is defined for $x \geqslant a$, we define the improper integral $\int_a^{\infty} f(x)\,dx = \lim_{b \to \infty} \int_a^b f(x)\,dx$, provided the limit exists.

4 When $f(x)$ is defined for $x \leqslant b$, we define the improper integral $\int_{-\infty}^{a} f(x)\,dx = \lim_{a \to -\infty} \int_a^b f(x)\,dx$, provided the limit exists.

5 When $f(x)$ is defined for $p < x < q$, but $f(x)$ is not defined when $x = p$, then the improper integral $\int_p^q f(x)\,dx = \lim_{a \to p+} \int_a^q f(x)\,dx$, provided the limit exists.

Numerical methods

Key point summary

1 If the graph of $y = f(x)$ is continuous over the interval $a \leqslant x \leqslant b$, and $f(a)$ and $f(b)$ have different signs, then a root of the equation $f(x) = 0$ must lie in the interval $a < x < b$.

2 When a root of $f(x) = 0$ is known to lie between $x = a$ and $x = b$, the bisection method requires you to next find $f\left(\frac{a+b}{2}\right)$. There must then be a change of sign which allows you to bisect the interval in which the root lies.
The procedure is repeated until you have an interval of the desired width containing the root.

3 When a root of the equation $f(x) = 0$ is known to lie between $x = a$ and $x = b$, linear interpolation involves replacing the curve by a straight line and gives an approximation to the root as

$$\frac{af(b) - bf(a)}{f(b) - f(a)}.$$

4 The Newton–Raphson iterative formula for solving $f(x) = 0$ is $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$.

5 Euler's method is used to find a numerical solution of the differential equation $\frac{dy}{dx} = f(x)$. The formula to find y is given by $y_{n+1} = y_n + hf(x_n)$, where $x_{n+1} = x_n + h$.

Linear laws

Key point summary

1 Since experimental data is not exact, due to measuring errors, points plotted are unlikely to all lie exactly in line so a line of best fit is drawn.

2 $y = mx + c$ is the equation of a straight line with gradient m and y-intercept c.

3 If the variables used are not x and y, the method to find the equation of the line of best fit is exactly the same. In the general equation $y = mx + c$ you just replace y by the variable on the vertical axis and replace x by the variable on the horizontal axis.

4 If the graph does not show the y-intercept, you can find the value of the gradient m as usual and then find the value of c by substituting the coordinates of a point on the line into the equation $y = mx + c$. Alternatively, you can use the coordinates of two points on the line to form and solve a pair of simultaneous equations in m and c.

5 To test a belief that the relation between x and y is of the form $y = ax^2 + b$, you need to plot y against x^2. If the points are roughly in a straight line, you can deduce that the relation between x and y is of the form $y = ax^2 + b$. The gradient of the line gives an estimate for a and the intercept on the vertical axis ($X = 0$) gives an estimate for b.

6 To test a belief that the relation between x and y is of the form $\frac{1}{x} + \frac{1}{y} = a$, you need to plot $\frac{1}{y}$ against $\frac{1}{x}$. If the points are roughly in a straight line with gradient -1, you can deduce that the relation between x and y is of the form $\frac{1}{x} + \frac{1}{y} = a$. The intercept on the vertical axis ($X = 0$) gives an estimate for a.

7 To test a belief that the relation between x and y is of the form $y = ax^2 + bx$, you can plot $\frac{y}{x}$ against x. If the points are roughly in a straight line, you can deduce that the relation between x and y is of the form $y = ax^2 + bx$. The gradient of the line gives an estimate for a and the intercept on the vertical axis ($X = 0$) gives an estimate for b.

8 Taking logarithms of both sides of $y = ax^n \Rightarrow \log y = \log a + n \log x$.

9 To test a belief that the relation between x and y is of the form $y = ax^n$, you need to plot $\log y$ against $\log x$. If the points are roughly in a straight line, you can deduce that the relation between x and y is of the form $y = ax^n$. The gradient of the line gives an estimate for n and the intercept on the vertical axis ($X = 0$) gives the value of $\log a$ from which the estimate for a can be found.

10 Taking logarithms of both sides of $y = ab^x \Rightarrow \log y = \log a + x \log b$.

11 To represent the relation $y = ab^x$ in a linear form you need to plot $\log y$ against x. If a straight line is obtained from the given data the relation is true. The gradient of the line is the value of $\log b$ and the intercept on the vertical axis ($X = 0$) gives the value of $\log a$. Knowing these two values, estimates for a and b can be found.

Trigonometry

Key point summary

1

θ in degrees	θ in radians	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	0	0	1	0
30	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90	$\frac{\pi}{2}$	1	0	∞
180	π	0	-1	0
360	2π	0	1	0

2 To find the general solution of $\sin x = k$ or $\cos x = k$, where $-1 \leqslant k \leqslant 1$, you find two solutions lying in the interval $-180° \leqslant x \leqslant 180°$ and then, since sine and cosine are periodic with period 360°, you add $360n°$, where n is an integer, to each of the two solutions.

3 For x in radians you find the general solution of $\sin x = k$ and $\cos x = k$, where $-1 \leqslant k \leqslant 1$ by finding two solutions lying in the interval $-\pi \leqslant x \leqslant \pi$ and then adding $2n\pi$, where n is an integer, to each of the two solutions.

4 The general solution of $\cos x = \cos\alpha$ is $x = 2n\pi \pm \alpha$ (or $x = (360n \pm \alpha)°$, if x is in degrees), where n is an integer.

5 The general solution of $\sin x = \sin\alpha$ is $x = 2n\pi + \alpha$, $2n\pi + \pi - \alpha$ (or $x = (360n + \alpha)°$, $(360n + 180 - \alpha)°$, if x is in degrees), where n is an integer.

6 To find the general solution of $\tan x = k$, where k is a constant, you find the solution lying in the interval $-90° \leqslant x \leqslant 90°$ and then, since tan is periodic with period 180°, you add $180n°$, where n is an integer, to the solution.

7 For x in radians you find the general solution of $\tan x = k$ by finding the solution lying in the interval

$-\frac{\pi}{2} \leqslant x \leqslant \frac{\pi}{2}$ then adding $n\pi$, where n is an integer,

to the solution.

8 The general solution of $\tan x = \tan\alpha$ is $x = n\pi + \alpha$ (or $x = (180n + \alpha)°$, if x is in degrees), where n is an integer.

Matrices and transformations

Key point summary

1 A **matrix** is a rectangular array of numbers. Each entry in the matrix is called an **element**.

2 A matrix with m rows and n columns is an $m \times n$ matrix. This is called the **order** of the matrix.

3 We can add or subtract matrices provided they have the **same order**.

4 To add/subtract matrices you add/subtract **corresponding elements**.

5 To multiply a matrix by a constant, simply multiply each element of the matrix by the constant.

6 We can multiply two matrices **A** and **B** only if the number of columns of **A** equals the number of rows of **B**.

7 If **A** is an $(a \times b)$ matrix and **B** is a $(c \times d)$ matrix then the product matrix **AB** exists if and only if $b = c$.
The product **AB** will be of order $a \times d$.

8 In general, $\mathbf{AB} \neq \mathbf{BA}$.
Matrix multiplication is not, in general, commutative.

9 A matrix which has the same number of rows and columns is called a **square matrix**.

10 The matrix $\mathbf{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is called the 2×2 **identity matrix** because when you multiply any 2×2 matrix **A** by **I** you get **A** as the answer.
This means that for any 2×2 matrix **A**,

$$\mathbf{IA} = \mathbf{AI} = \mathbf{A}.$$

11 The 2×2 matrix $\mathbf{Z} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ is called the **zero matrix** p48

since $\mathbf{Z} + \mathbf{A} = \mathbf{A} + \mathbf{Z} = \mathbf{A}$

and $\mathbf{ZA} = \mathbf{AZ} = \mathbf{Z}$ for any 2×2 matrix **A**.

Matrix transformations

Key point summary

1 A linear transformation that changes/transforms point $P(x, y)$ into point $P'(x', y')$ can be written as:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \qquad \textbf{matrix form}$$

or $\quad x' = ax + by \qquad$ **algebraic form**

$\quad\;\; y' = cx + dy$

2 The matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is called a transformation matrix.

3 A one-way stretch in the x-direction of scale factor k is given by:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} k & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}.$$

4 A one-way stretch in the y-direction of scale factor k is given by:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & k \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}.$$

5 A two-way stretch of scale factor a in the x-direction and scale factor b in the y-direction is given by:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}.$$

6 An enlargement of scale factor k with centre the origin is given by:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}.$$

7 A rotation through angle θ anticlockwise about the origin is given by:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}.$$

8 A reflection in the line $y = x\tan\theta$ (where θ is the angle the line makes with the positive x-axis) is given by:

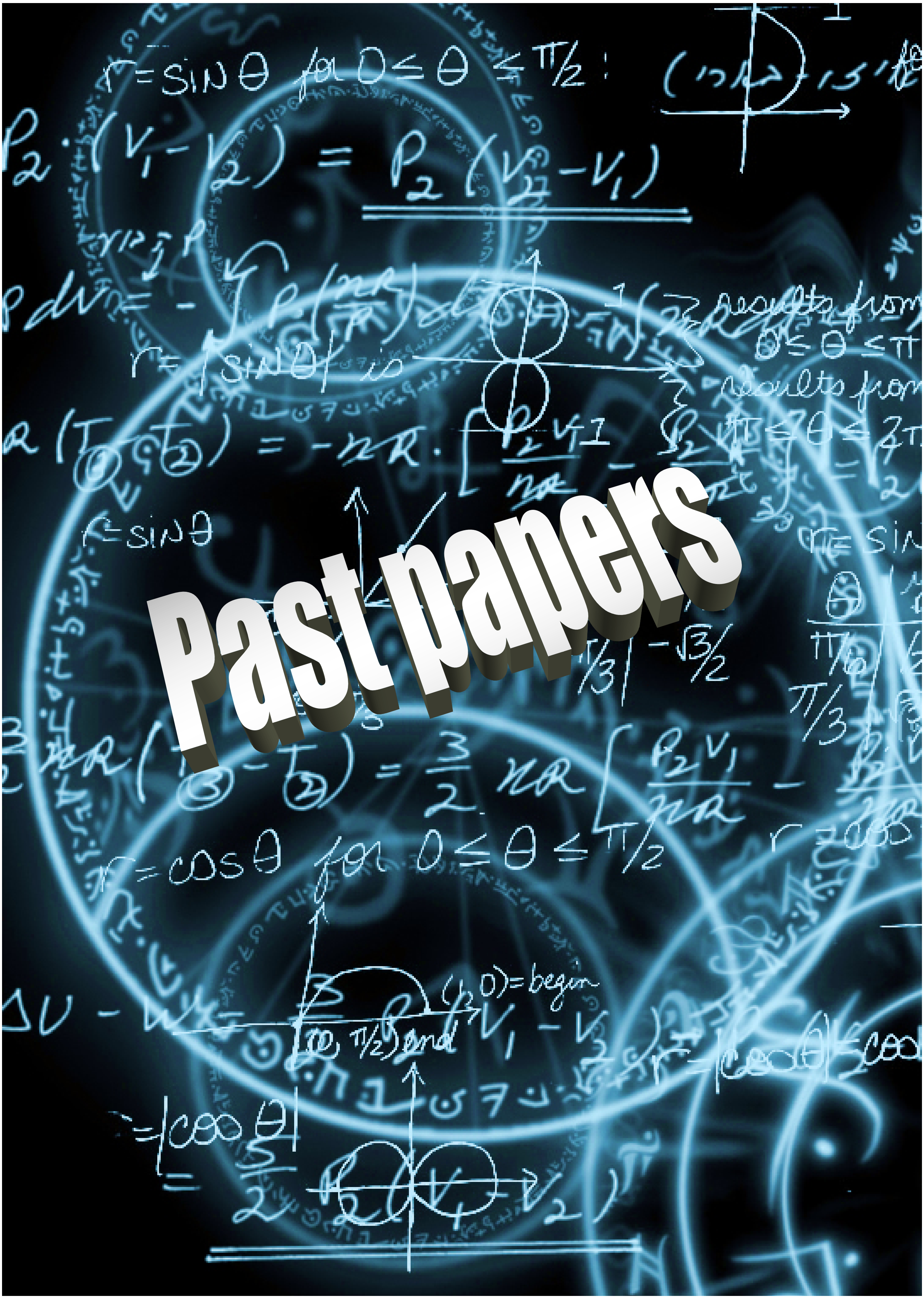
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}.$$

9 If the 2×2 matrices, **A** and **B**, both represent transformations, then the product matrix **BA** also represents a transformation equivalent to applying **A** followed by **B**.

This type of transformation is known as a composite transformation.

It is important to remember to read composite transformations backwards, i.e.,

ABC means **C** first then **B** then **A**.


Past papers

General Certificate of Education
January 2006
Advanced Subsidiary Examination

MATHEMATICS **MFP1**
Unit Further Pure 1

Monday 23 January 2006 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables
- an insert for use in Question 6 (enclosed)

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP1.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- Fill in the boxes at the top of the insert.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 (a) Show that the equation

$$x^3 + 2x - 2 = 0$$

has a root between 0.5 and 1. *(2 marks)*

(b) Use linear interpolation once to find an estimate of this root. Give your answer to two decimal places. *(3 marks)*

2 (a) For each of the following improper integrals, find the value of the integral **or** explain briefly why it does not have a value:

(i) $\displaystyle\int_0^9 \frac{1}{\sqrt{x}}\,dx$; *(3 marks)*

(ii) $\displaystyle\int_0^9 \frac{1}{x\sqrt{x}}\,dx$. *(3 marks)*

(b) Explain briefly why the integrals in part (a) are improper integrals. *(1 mark)*

3 Find the general solution, in **degrees**, for the equation

$$\sin(4x + 10^\circ) = \sin 50^\circ$$ *(5 marks)*

4 A curve has equation

$$y = \frac{6x}{x-1}$$

(a) Write down the equations of the two asymptotes to the curve. *(2 marks)*

(b) Sketch the curve and the two asymptotes. *(4 marks)*

(c) Solve the inequality

$$\frac{6x}{x-1} < 3$$ *(4 marks)*

5 (a) (i) Calculate $(2 + \mathrm{i}\sqrt{5})(\sqrt{5} - \mathrm{i})$. *(3 marks)*

(ii) Hence verify that $\sqrt{5} - \mathrm{i}$ is a root of the equation

$$(2 + \mathrm{i}\sqrt{5})\,z = 3z^*$$

where z^* is the conjugate of z. *(2 marks)*

(b) The quadratic equation

$$x^2 + px + q = 0$$

in which the coefficients p and q are real, has a complex root $\sqrt{5} - \mathrm{i}$.

(i) Write down the other root of the equation. *(1 mark)*

(ii) Find the sum and product of the two roots of the equation. *(3 marks)*

(iii) Hence state the values of p and q. *(2 marks)*

6 [**Figure 1** and **Figure 2**, printed on the insert, are provided for use in this question.]

The variables x and y are known to be related by an equation of the form

$$y = kx^n$$

where k and n are constants.

Experimental evidence has provided the following approximate values:

x	4	17	150	300
y	1.8	5.0	30	50

(a) Complete the table in **Figure 1**, showing values of X and Y, where

$$X = \log_{10} x \quad \text{and} \quad Y = \log_{10} y$$

Give each value to two decimal places. *(3 marks)*

(b) Show that if $y = kx^n$, then X and Y must satisfy an equation of the form

$$Y = aX + b$$ *(3 marks)*

(c) Draw on **Figure 2** a linear graph relating X and Y. *(3 marks)*

(d) Find an estimate for the value of n. *(2 marks)*

7 (a) The transformation T is defined by the matrix $\mathbf{A}$, where

$$\mathbf{A} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

(i) Describe the transformation T geometrically. *(2 marks)*

(ii) Calculate the matrix product $\mathbf{A}^2$. *(2 marks)*

(iii) Explain briefly why the transformation T followed by T is the identity transformation. *(1 mark)*

(b) The matrix $\mathbf{B}$ is defined by

$$\mathbf{B} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

(i) Calculate $\mathbf{B}^2 - \mathbf{A}^2$. *(3 marks)*

(ii) Calculate $(\mathbf{B} + \mathbf{A})(\mathbf{B} - \mathbf{A})$. *(3 marks)*

8 A curve has equation $y^2 = 12x$.

(a) Sketch the curve. *(2 marks)*

(b) (i) The curve is translated by 2 units in the positive y direction. Write down the equation of the curve after this translation. *(2 marks)*

(ii) The **original** curve is reflected in the line $y = x$. Write down the equation of the curve after this reflection. *(1 mark)*

(c) (i) Show that if the straight line $y = x + c$, where c is a constant, intersects the curve $y^2 = 12x$, then the x-coordinates of the points of intersection satisfy the equation

$$x^2 + (2c - 12)x + c^2 = 0$$ *(3 marks)*

(ii) Hence find the value of c for which the straight line is a tangent to the curve. *(2 marks)*

(iii) Using this value of c, find the coordinates of the point where the line touches the curve. *(2 marks)*

(iv) In the case where $c = 4$, determine whether the line intersects the curve or not. *(3 marks)*

END OF QUESTIONS

MFP1

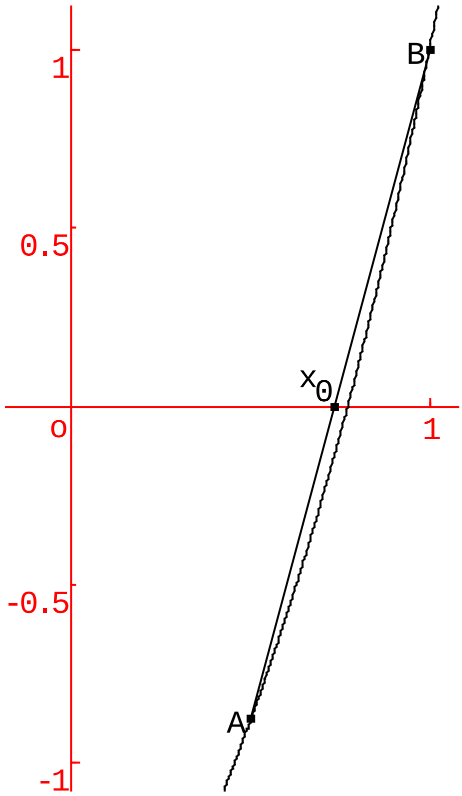
Q	Solution	Marks	Totals	Comments
1(a)	$f(0.5) = -0.875$, $f(1) = 1$	B1		
	Change of sign, so root between	E1	2	
(b)	Complete line interpolation method	M2,1		M1 for partially correct method
	Estimated root = $\frac{11}{15} \approx 0.73$	A1	3	Allow $\frac{11}{15}$ as answer
	Total		**5**	
2(a)(i)	$\int x^{-\frac{1}{2}}\,dx = 2x^{\frac{1}{2}}\ (+c)$	M1A1		M1 for $kx^{\frac{1}{2}}$
	$\int_0^9 \frac{1}{\sqrt{x}}\,dx = 6$	A1√	3	ft wrong coeff of $x^{\frac{1}{2}}$
(ii)	$\int x^{-\frac{1}{2}}\,dx = -2x^{-\frac{1}{2}}\ (+c)$	M1A1		M1 for $kx^{-\frac{1}{2}}$
	$x^{-\frac{1}{2}} \to \infty$ as $x \to 0$, so no value	E1	3	'Tending to infinity' clearly implied
(b)	Denominator $\to 0$ as $x \to 0$	E1	1	
	Total		**7**	
3	One solution is $x = 10°$	B1		PI by general formula
	Use of sin 130º = sin 50º	M1		OE
	Second solution is $x = 30°$	A1		OE
	Introduction of $90n$º, or $360n$º or $180n$º	M1		Or $\pi n/2$ or $2\pi n$ or πn
	GS $(10+90n)°, (30+90n)°$	A1√	5	OE; ft one numerical error or omission of 2nd soln
	Total		**5**	
4(a)	Asymptotes $x = 1$, $y = 6$	B1B1	2	
(b)	Curve (correct general shape)	M1		SC Only one branch:
	Curve passing through origin	A1		B1 for origin
	Both branches approaching $x = 1$	A1		B1 for approaching both asymptotes
	Both branches approaching $y = 6$	A1	4	(Max 2/4)
(c)	Correct method	M1		
	Critical values ± 1	B1B1		From graph or calculation
	Solution set $-1 < x < 1$	A1√	4	ft one error in CVs; NMS 4/4 after a good graph
	Total		**10**	
5(a)(i)	Full expansion of product	M1		
	Use of $i^2 = -1$	m1		
	$(2+\sqrt{5}i)(\sqrt{5}-i) = 3\sqrt{5}+3i$	A1	3	$\sqrt{5}\sqrt{5} = 5$ must be used − Accept not fully simplified
(ii)	$z^* = x - iy\ (= \sqrt{5} + i)$	M1		
	Hence result	A1	2	Convincingly shown (AG)
(b)(i)	Other root is $\sqrt{5} + i$	B1	1	
(ii)	Sum of roots is $2\sqrt{5}$	B1		
	Product is 6	M1A1	3	
(iii)	$p = -2\sqrt{5}$, $q = 6$	B1 B1√	2	ft wrong answers in (ii)
	Total		**11**	

MFP1

Q	Solution	Marks	Totals	Comments
6(a)	X values 1.23, 2.18			
	Y values 0.70, 1.48	B3,2,1	3	−1 for each error
(b)	$\lg y = \lg k + \lg x^n$	M1		
	$\lg x^n = n \lg x$	M1		
	So $Y = nX + \lg k$	A1	3	
(c)	Four points plotted	B2,1√		B1 if one error here; ft wrong values in (a)
	Good straight line drawn	B1√	3	ft incorrect points (approx collinear)
(d)	Method for gradient	M1		
	Estimate for n	A1√	2	Allow AWRT 0.75 - 0.78; ft grad of candidate's graph
	Total		**11**	
7(a)(i)	Reflection ...	M1		
	... in $y = -x$	A1	2	OE
(ii)	$\mathbf{A}^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	M1A1	2	M1A0 for three correct entries
(iii)	$\mathbf{A}^2 = \mathbf{I}$ or geometrical reasoning	E1	1	
(b)(i)	$\mathbf{B}^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$	M1A1		M1A0 for three correct entries
	$\mathbf{B}^2 - \mathbf{A}^2 = \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}$	A1√	3	ft errors, dependent on both M marks
(ii)	$(\mathbf{B}+\mathbf{A})(\mathbf{B}-\mathbf{A}) = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$	B1		
	$\ldots = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$	M1 A1√	3	ft one error; M1A0 for three correct (ft) entries
	Total		**11**	
8(a)	Good attempt at sketch	M1		
	Correct at origin	A1	2	
(b)(i)	y replaced by $y - 2$	B1		
	Equation is $(y-2)^2 = 12x$	B1√	2	ft $y + 2$ for $y - 2$
(ii)	Equation is $x^2 = 12y$	B1	1	
(c)(i)	$(x+c)^2 = x^2 + 2cx + c^2$	B1		
	$\ldots = 12x$	M1		
	Hence result	A1	3	convincingly shown (AG)
(ii)	Tangent if $(2c-12)^2 - 4c^2 = 0$	M1		
	ie if $-48c + 144 = 0$ so $c = 3$	A1	2	
(iii)	$x^2 - 6x + 9 = 0$	M1		
	$x = 3, y = 6$	A1	2	
(iv)	$c = 4 \Rightarrow$ discriminant $= -48 < 0$	M1A1		OE
	So line does not intersect curve	A1	3	
	Total		**15**	
	TOTAL		**75**	

Further pure 1 - AQA - January 2006

Question 1



$1)\ Call\ f(x)=x^3+2x-2$

$a)\qquad f(0.5)=0.5^3+2\times0.5-2=-0.875<0$

$\qquad f(1)=1^3+2\times1-1=1>0$

According to the sign change rule,

we can say that there is at least a root of f in the interval $[0.5,1]$

$b)$ Let's call A(0.5,f(0.5)) and B(1,f(1))

The gradient of the line AB is $\dfrac{f(1)-f(0.5)}{1-0.5}=\dfrac{15}{4}$

An equation of the line AB is : $y-1=\dfrac{15}{4}(x-1)$

This line croos the $x-$axis when $y=0$

$$which\ \ gives\ -1=\frac{15}{4}(x-1)$$

$$\frac{-4}{15}=x-1$$

$$x=\frac{11}{15}(\approx 0.73)$$

Question 2:

$2)\,a)\,i)\int \dfrac{1}{\sqrt{x}}dx=\int x^{-\frac{1}{2}}dx=2\sqrt{x}+c \qquad f(x)=2\sqrt{x}\text{'s domain is }\mathbb{R},$

we can therefore work out $\displaystyle\int_0^9\frac{1}{\sqrt{x}}dx=\left[2\sqrt{x}\right]_0^9=\left(2\sqrt{9}\right)-\left(2\sqrt{0}\right)=6$

$ii)\int\dfrac{1}{x\sqrt{x}}dx=\int x^{-\frac{3}{2}}dx=-2x^{-\frac{1}{2}}=\dfrac{-2}{\sqrt{x}} \qquad when\ \ x=0,\ \dfrac{-2}{\sqrt{x}}\ is\ \ not\ \ defined.$

This integral does not have a value.

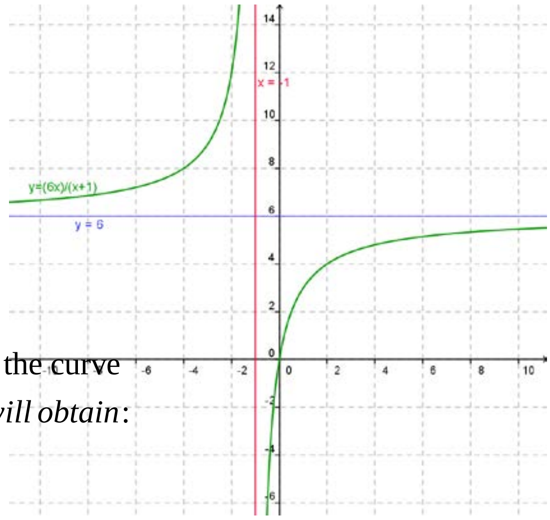
$b)$ These integrals are improper because the function to integrate are not defined for $x=0$.

Question 3:

$Sin(4x+10)=Sin(50)$

so	$4x+10=50+360n$	$4x=40+360n$	$x=10^o+90^o n$
or	$4x+10=(180-50)+360n$	$4x=120+360n$	$x=30^o+90^o n$
			$n\in\mathbb{N}$

Question 4:



a) • $y=\frac{6x}{x+1}$, *dividing by x gives* $y=\frac{6}{1+\frac{1}{x}}$

when $x\to\infty, y\to 6$ *because* $\frac{1}{x}\to 0$

The line with equation $y=6$ is asymptote to the curve

• *When* $x\to -1,\ y\to\infty$

the line with equation $x=-1$ is a (vertical) asymptote to the curve

b) *With a table of values (to plot a couple of "guide" points) we will obtain*:

c) $\frac{6x}{x-1}<3$ *We* solve $\frac{6x}{x-1}=3$ $6x=3(x-1)$

$3x=-3$ $x=-1.$

On the graph, draw the line y =3 .

From this, we can conclude that the solutions of $\frac{6x}{x-1}<3$

are all the values x *with* $-1<x<1$

Question 5:

a) i) $\left(2+i\sqrt{5}\right)\left(\sqrt{5}-i\right)=2\sqrt{5}-2i+5i-i^2\sqrt{5}=3\sqrt{5}+3i$

ii) *if* $z=\sqrt{5}-i$ *then* $z^*=\sqrt{5}+i$ *and* $3z^*=3\sqrt{5}+3i$

so indeed $(2+i\sqrt{5})z=3z^*$

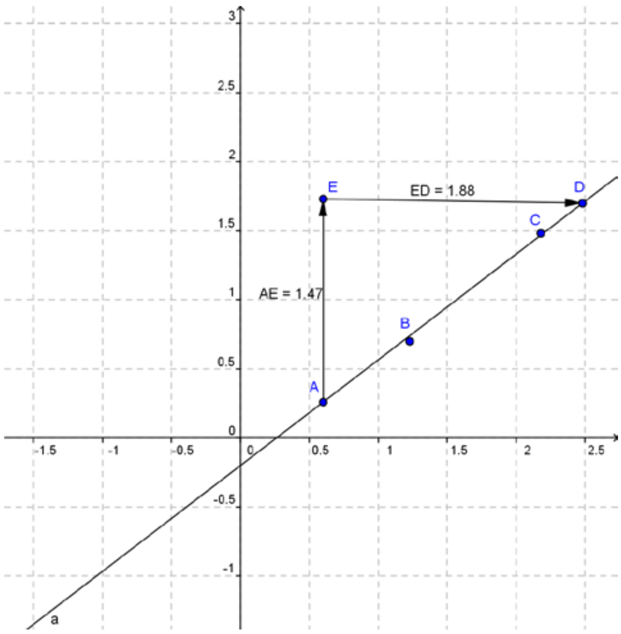
b) i) If one complex root is $\sqrt{5}-i$, then the other root is the conjugate $\sqrt{5}+i$

ii) *Sum*: $\sqrt{5}+i+\sqrt{5}-1=2\sqrt{5}$ Product: $(\sqrt{5}+i)(\sqrt{5}-i)=6$

iii) *therefore* $q=6$ *and* $p=-2\sqrt{5}$

Question 6:

X	0.60	1.23	2.18	2.48
Y	0.26	0.70	1.48	1.70



b) $y=kx^n$ *then* $\log_{10}(y)=\log_{10}(kx^n)$

$Y=\log_{10}k+n\log_{10}(x)$

$Y=nX+\log_{10}k$

$Y=aX+b$

with $a=n$ *and* $b=\log_{10}k$

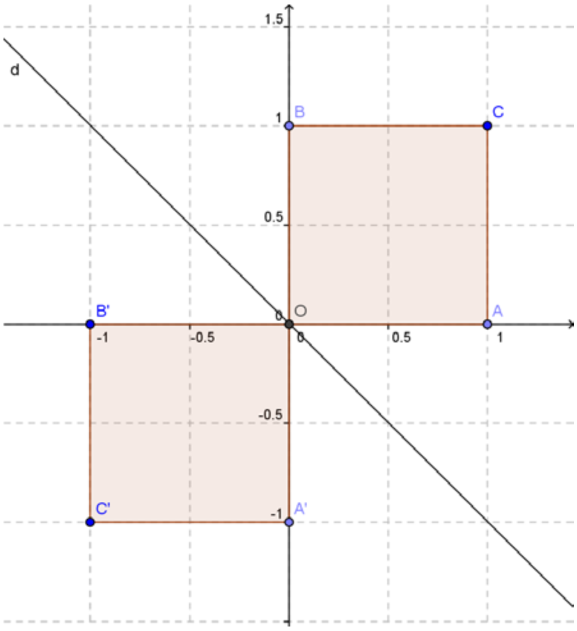
c) An estimate of the gradient is $n=\frac{1.47}{1.88}=0.78$

Question 7:

$i) T$ is the reflection in the line $y=-x$

$$ii)\ A^2=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$iii)$ The reflection in $y=-x$ followed by the same reflection return the same object you started with.



$$\text{b)}\ i) B=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix} \qquad B^2=\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}$$

$$and\ \ B^2-A^2=\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}-\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}0 & 2\\ 0 & 0\end{bmatrix}$$

$$ii)\ (B+A)(B-A)=\begin{bmatrix}1 & 0\\ -1 & 1\end{bmatrix}\times\begin{bmatrix}1 & 2\\ 1 & 1\end{bmatrix}=\begin{bmatrix}1 & 2\\ 0 & -1\end{bmatrix}$$

Question 8:

$a)$

$b) i) (y-2)^2=12x$

$ii)\ x^2=12y$

$$c)\ i)\begin{cases} y^2=12x\\ y=x+c\end{cases} \qquad gives\ \ (x+c)^2=12x$$

$$x^2+2cx+c^2-12x=0$$

$$x^2+(2c-12)x+c^2=0 \qquad (E)$$

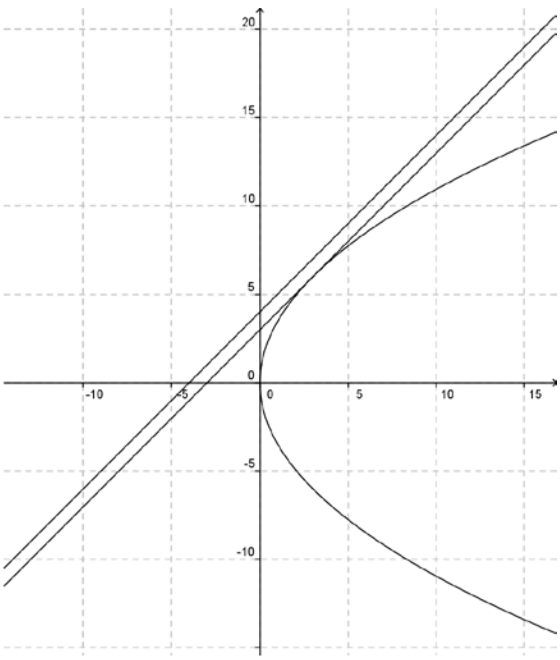
$ii)$ The line is tangent to the curve when there is one unique solution to this equation meaning that the discriminant is equal to 0

$$\text{Discriminant: } (2c-12)^2-4\times1\times c^2=0$$

$$4c^2-48c+144-4c^2=0$$

$$-48c+144=0$$

$$c=3$$



$iii)$ *When* $c=3$, *the equation* (E) *becomes*

$$x^2-6x+9=0$$

$$(x-3)^2=0 \qquad so\ \ x=3$$

$$and\ \ y=x+c=6$$

The coordinates of the point where the line and the curve touches: $(3,6)$

$iv)$ *When* $c=4$, *the equation* (E) *becomes* $x^2-4x+16=0$

The discriminant is $(-4)^2-4\times1\times16=-48<0$

The line and the curve do not intersect.

Grade boundaries

Comp. Code	Component Title	Maximum Scaled Mark	A	B	C	D	E
			Scaled Mark Grade Boundaries				
MFP1	GCE MATHEMATICS UNIT FP1	75	62	54	46	39	32

General Certificate of Education
June 2006
Advanced Subsidiary Examination

MATHEMATICS **MFP1**
Unit Further Pure 1

Monday 12 June 2006 1.30 pm to 3.00 pm

For this paper you must have: • an 8-page answer book • the **blue** AQA booklet of formulae and statistical tables You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The quadratic equation

$$3x^2 - 6x + 2 = 0$$

has roots α and β.

(a) Write down the numerical values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) (i) Expand $(\alpha + \beta)^3$. *(1 mark)*

(ii) Show that $\alpha^3 + \beta^3 = 4$. *(3 marks)*

(c) Find a quadratic equation with roots α^3 and β^3, giving your answer in the form $px^2 + qx + r = 0$, where p, q and r are integers. *(3 marks)*

2 A curve satisfies the differential equation

$$\frac{dy}{dx} = \log_{10} x$$

Starting at the point (2, 3) on the curve, use a step-by-step method with a step length of 0.2 to estimate the value of y at $x = 2.4$. Give your answer to three decimal places. *(6 marks)*

3 Show that

$$\sum_{r=1}^{n} (r^2 - r) = kn(n+1)(n-1)$$

where k is a rational number. *(4 marks)*

4 Find, in **radians**, the general solution of the equation

$$\cos 3x = \frac{\sqrt{3}}{2}$$

giving your answers in terms of π. *(5 marks)*

5 The matrix $\mathbf{M}$ is defined by

$$\mathbf{M} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

(a) Find the matrix:

(i) $\mathbf{M}^2$; *(3 marks)*

(ii) $\mathbf{M}^4$. *(1 mark)*

(b) Describe fully the geometrical transformation represented by $\mathbf{M}$. *(2 marks)*

(c) Find the matrix $\mathbf{M}^{2006}$. *(3 marks)*

6 It is given that $z = x + \mathrm{i}y$, where x and y are real numbers.

(a) Write down, in terms of x and y, an expression for

$$(z + \mathrm{i})^*$$

where $(z + \mathrm{i})^*$ denotes the complex conjugate of $(z + \mathrm{i})$. *(2 marks)*

(b) Solve the equation

$$(z + \mathrm{i})^* = 2\mathrm{i}z + 1$$

giving your answer in the form $a + b\mathrm{i}$. *(5 marks)*

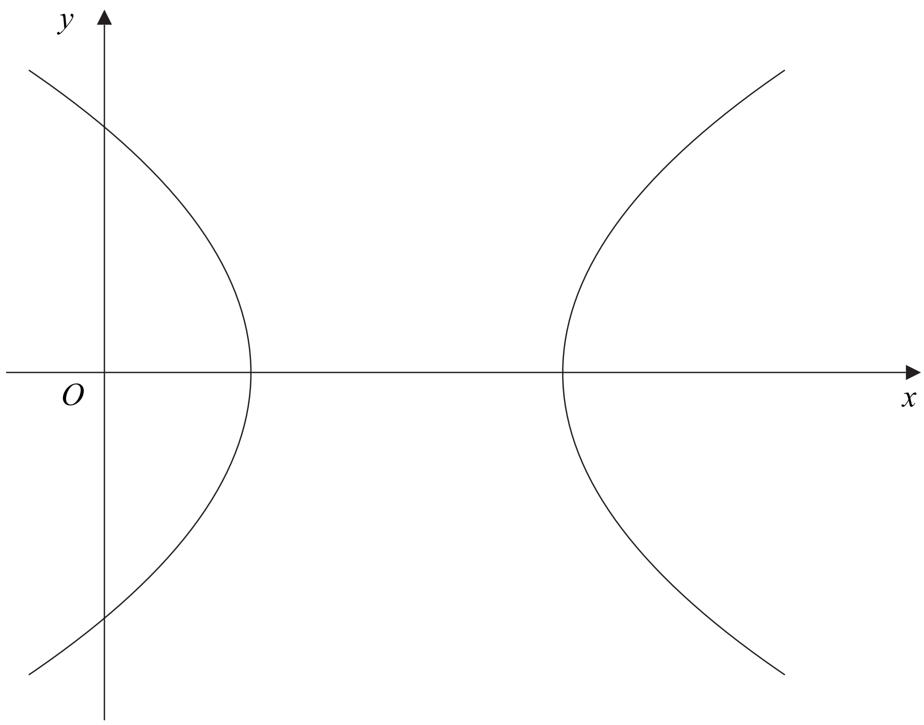
7 (a) Describe a geometrical transformation by which the hyperbola

$$x^2 - 4y^2 = 1$$

can be obtained from the hyperbola $x^2 - y^2 = 1$. *(2 marks)*

(b) The diagram shows the hyperbola H with equation

$$x^2 - y^2 - 4x + 3 = 0$$



By completing the square, describe a geometrical transformation by which the hyperbola H can be obtained from the hyperbola $x^2 - y^2 = 1$. *(4 marks)*

8 (a) The function f is defined for all real values of x by

$$\mathrm{f}(x) = x^3 + x^2 - 1$$

(i) Express $\mathrm{f}(1+h) - \mathrm{f}(1)$ in the form

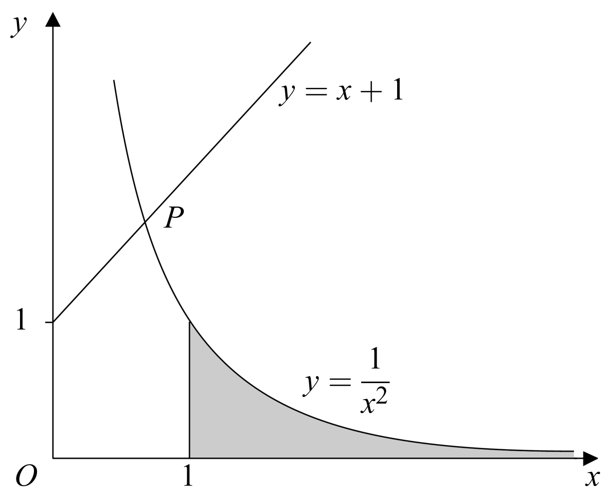
$$ph + qh^2 + rh^3$$

where p, q and r are integers. *(4 marks)*

(ii) Use your answer to part (a)(i) to find the value of $\mathrm{f}'(1)$. *(2 marks)*

(b) The diagram shows the graphs of

$$y = \frac{1}{x^2} \quad \text{and} \quad y = x + 1 \quad \text{for} \quad x > 0$$



The graphs intersect at the point P.

(i) Show that the x-coordinate of P satisfies the equation $\mathrm{f}(x) = 0$, where f is the function defined in part (a). *(1 mark)*

(ii) Taking $x_1 = 1$ as a first approximation to the root of the equation $\mathrm{f}(x) = 0$, use the Newton–Raphson method to find a second approximation x_2 to the root. *(3 marks)*

(c) The region enclosed by the curve $y = \dfrac{1}{x^2}$, the line $x = 1$ and the x-axis is shaded on the diagram. By evaluating an improper integral, find the area of this region. *(3 marks)*

9 A curve C has equation

$$y = \frac{(x+1)(x-3)}{x(x-2)}$$

(a) (i) Write down the coordinates of the points where C intersects the x-axis. *(2 marks)*

(ii) Write down the equations of all the asymptotes of C. *(3 marks)*

(b) (i) Show that, if the line $y = k$ intersects C, then

$$(k-1)(k-4) \geqslant 0$$ *(5 marks)*

(ii) Given that there is only one stationary point on C, find the coordinates of this stationary point.

(No credit will be given for solutions based on differentiation.) *(3 marks)*

(c) Sketch the curve C. *(3 marks)*

END OF QUESTIONS

MFP1

Q	Solution	Marks	Total	Comments
1(a)	$\alpha+\beta=2,\ \alpha\beta=\frac{2}{3}$	B1B1	2	**SC** 1/2 for answers 6 and 2
(b)(i)	$(\alpha+\beta)^3=\alpha^3+3\alpha^2\beta+3\alpha\beta^2+\beta^3$	B1	1	Accept unsimplified
(ii)	$\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$	M1		
	Substitution of numerical values	m1		
	$\alpha^3+\beta^3=4$	A1	3	convincingly shown AG
(c)	$\alpha^3\beta^3=\frac{8}{27}$	B1		
	Equation of form $px^2\pm 4px+r=0$	M1		
	Answer $27x^2-108x+8=0$	A1√	3	ft wrong value for $\alpha^3\beta^3$
	Total		**9**	
2	1st increment is 0.2 lg 2 ...	M1		or 0.2 lg 2.1 or 0.2 lg 2.2
	... ≈ 0.06021	A1		PI
	$x=2.2\Rightarrow y\approx 3.06021$	A1√		PI; ft numerical error
	2nd increment is 0.2 lg 2.2	m1		consistent with first one
	... ≈ 0.06848	A1		PI
	$x=2.4\Rightarrow y\approx 3.12869\approx 3.129$	A1√	6	ft numerical error
	Total		**6**	
3	$\Sigma(r^2-r)=\Sigma r^2-\Sigma r$	M1		
	At least one linear factor found	m1		
	$\Sigma(r^2-r)=\frac{1}{6}n(n+1)(2n+1-3)$	m1		OE
	$\ldots=\frac{1}{3}n(n+1)(n-1)$	A1	4	
	Total		**4**	
4	$\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}$ stated or used	B1		Condone decimals and/or degrees until final mark
	Appropriate use of ±	B1		
	Introduction of $2n\pi$	M1		
	Division by 3	M1		Of $\alpha+kn\pi$ or $\pm\alpha+kn\pi$
	$x=\pm\frac{\pi}{18}+\frac{2}{3}n\pi$	A1	5	
	Total		**5**	
5(a)(i)	$\mathbf{M}^2=\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}$	M1 A2,1	3	M1 if 2 entries correct M1A1 if 3 entries correct
(ii)	$\mathbf{M}^4=\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$	B1√	1	ft error in $\mathbf{M}^2$ provided no surds in $\mathbf{M}^2$
(b)	Rotation (about the origin)	M1		
	... through 45° clockwise	A1	2	
(c)	Awareness of $\mathbf{M}^8=\mathbf{I}$	M1		OE; NMS 2/3
	$\mathbf{M}^{2006}=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$	m1 A1√	3	complete valid method ft error in $\mathbf{M}^2$ as above
	Total		**9**	

MFP1 (cont)

Q	Solution	Marks	Total	Comments
6(a)	$(z+\mathrm{i})^* = x - \mathrm{i}y - \mathrm{i}$	B2	2	
(b)	$\ldots = 2\mathrm{i}x - 2y + 1$	M1		$\mathrm{i}^2 = -1$ used at some stage
	Equating R and I parts	M1		involving at least 5 terms in all
	$x = -2y + 1,\ -y - 1 = 2x$	A1√		ft one sign error in (a)
	$z = -1 + \mathrm{i}$	m1A1√	5	ditto; allow $x = -1$, $y = 1$
	Total		**7**	
7(a)	Stretch parallel to y axis ...	B1		
	... scale-factor $\frac{1}{2}$ parallel to y axis	B1	2	
(b)	$(x-2)^2 - y^2 = 1$	M1A1		
	Translation in x direction ...	A1		
	... 2 units in positive x direction	A1	4	
	Total		**6**	
8(a)(i)	$(1+h)^3 = 1 + 3h + 3h^2 + h^3$	B1		
	$\mathrm{f}(1+h) = 1 + 5h + 4h^2 + h^3$	M1A1√		PI; ft wrong coefficients
	$\mathrm{f}(1+h) - \mathrm{f}(1) = 5h + 4h^2 + h^3$	A1√	4	ft numerical errors
(ii)	Dividing by h	M1		
	$\mathrm{f}'(1) = 5$	A1√	2	ft numerical errors
(b)(i)	$x^2(x+1) = 1$, hence result	B1	1	convincingly shown (AG)
(ii)	$x_2 = 1 - \frac{1}{5} = \frac{4}{5}$	M1A1√ A1√	3	ft c's value of f′(1)
(c)	Area $= \int_1^\infty x^{-2}\,\mathrm{d}x$	M1		
	$\ldots = \left[-x^{-1}\right]_1^\infty$	M1		Ignore limits here
	$\ldots = 0 - -1 = 1$	A1	3	
	Total		**13**	
9(a)(i)	Intersections at $(-1, 0)$, $(3, 0)$	B1B1	2	Allow $x = -1$, $x = 3$
(ii)	Asymptotes $x = 0$, $x = 2$, $y = 1$	B1 × 3	3	
(b)(i)	$y = k \Rightarrow kx^2 - 2kx = x^2 - 2x - 3$	M1A1		M1 for clearing denominator
	$\ldots \Rightarrow (k-1)x^2 + (-2k+2)x + 3 = 0$	A1√		ft numerical error
	$\Delta = 4(k-1)(k-4)$, hence result	m1A1	5	convincingly shown (AG)
(ii)	$y = 4$ at SP	B1		
	$3x^2 - 6x + 3 = 0$, so $x = 1$	M1A1	3	A0 if other point(s) given
(c)	Curve with three branches	B1		approaching vertical asymptotes
	Middle branch correct	B1		Coordinates of SP not needed
	Other two branches correct	B1	3	3 asymptotes shown
	Total		**16**	
	TOTAL		**75**	

Further pure 1 - AQA - June 2006

Question 1:

$1)\, 3x^2 - 6x + 2 = 0 \quad has\ roots\ \alpha\ and\ \beta$

$$a)\ \alpha + \beta = \frac{6}{3} = 2 \qquad \alpha\beta = \frac{2}{3}$$

$$b)\, i) (\alpha+\beta)^3 = \alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3$$

$$ii)\ \alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha^2\beta - 3\alpha\beta^2 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$$

$$\alpha^3 + \beta^3 = 2^3 - 3\times\frac{2}{3}\times 2 = 8 - 4 = 4$$

$$c)\,\text{If } u = \alpha^3 \ \ and\ \ v = \beta^3 \qquad then$$

$$u + v = 4 \ \ and\ \ uv = \alpha^3\beta^3 = (\alpha\beta)^3 = \frac{8}{27}$$

An equation with roots $u = \alpha^3$ and $v = \beta^3$ is : $x^2 - 4x + \frac{8}{27} = 0$

$$27x^2 - 108x + 8 = 0$$

Question 2:

$Euler\ \ formula : y_{n+1} = y_n + hf(x_n)$

$$x_1 = 2 \ \ and\ \ h = 0.2$$

$for\ \ x_2 = 2.2\ ,\ \ y_2 \approx 3 + 0.2\times\log_{10}(2) \approx 3.0602$

$for\ \ x_3 = 2.4\ ,\ \ y_3 \approx 3.0602 + 0.2\times\log_{10}(2.2) \approx 3.129$

Question 3:

$$\sum_{r=1}^{n}(r^2 - r) = \sum_{r=1}^{n} r^2 - \sum_{r=1}^{n} r = \frac{1}{6}n(n+1)(2n+1) - \frac{1}{2}n(n+1)$$

$$= \frac{1}{6}n(n+1)[2n+1-3] = \frac{1}{3}n(n+1)(n-1)$$

Question 4:

$$Cos3x = \frac{\sqrt{3}}{2}$$

$$3x = \frac{\pi}{6} + k2\pi \quad or \quad 3x = -\frac{\pi}{6} + k2\pi$$

$$x = \frac{\pi}{18} + k\frac{2\pi}{3} \quad or \quad x = -\frac{\pi}{18} + k\frac{2\pi}{3} \qquad k \in \mathbb{Z}$$

Question 5:

$$M=\begin{bmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{bmatrix} \qquad a)i)\; M^2=\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}$$

$$M^4=\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$$

$b)$ M represents the rotation centre (0,0) angle $\frac{\pi}{4}(=45^o)$ *clockwise*

$c)$ *Because* M represents a rotation with angle 45^0 *then* M^8 is a rotation with angle $45\times8=360^o$

$M^8=I$

$$M^{2006}=M^{8\times250+6}=\left(M^8\right)^{250}\times M^6=I^{250}\times M^6=M^4\times M^2$$

$$M^{2006}=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$$

Question 6:

$a)\; z=x+iy$

$$(z+i)^*=(x+i(y+1))^*=x-i(y+1)$$

$b)\; (z+i)^*=2iz+1$

$x-i(y+1)=2i(x+iy)+1$

$x-i(y+1)=2ix-2y+1$

$x+i(-1-y)=1-2y+2ix$ *therefore*

$x=1-2y$ Real parts are equal

$-1-y=2x$ Imaginary parts are equal

We solve

$-1-y=2(1-2y)$

$1-y=2-4y$

$3y=3 \qquad y=1$

and $x=1-2y=1-2=-1$

$$z=-1+i$$

Question 7:

$a)\; x^2-4y^2=1$

$x^2-(2y)^2=1$

This hyperbola can be obtain by a stretch, scale factor $\frac{1}{2}$, parallel to the y-axis of the hyperbola $x^2-y^2=1$

$b)\; x^2-y^2-4x+3=0$

$x^2-4x-y^2+3=0$

$(x-2)^2-4-y^2+3=0$

$(x-2)^2-y^2=1$

This hyperbola can be obtain by a *translation of vector* $\begin{pmatrix}2\\0\end{pmatrix}$ of the hyperbola $x^2-y^2=1$

Question 8:

a) $f(x) = x^3 + x^2 - 1$

$$i)\ f(1+h) - f(1) = \left((1+h)^3 + (1+h)^2 - 1\right) - \left(1^3 + 1^2 - 1\right)$$

$$= \left(1 + 3h + 3h^2 + h^3 + 1 + 2h + h^2 - 1\right) - (1)$$

$$= 5h + 4h^2 + h^3$$

$$ii)\ \frac{f(1+h) - (1)}{h} = \frac{5h + 4h^2 + h^3}{h} = 5 + 4h + h^2 \xrightarrow[h \to 0]{} 5$$

$$so\ \ f'(1) = 5$$

b) i) The x-coordinate of P satisfy the equation $\frac{1}{x^2} = x + 1$

$$multiply\ by\ x^2: \qquad 1 = x^3 + x^2$$

$$x^3 + x^2 - 1 = 0$$

$$f(x) = 0$$

ii) If x_1 is an approximation of the root then $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$ is a better approximation.

$x_1 = 1,\ \ f(x_1) = f(1) = 1\ and\ \ f'(1) = 5$

$$so \qquad x_2 = 1 - \frac{1}{5} = \frac{4}{5} = 0.8$$

c) If it exits, the area is $\int_1^{\infty} \frac{1}{x^2}dx. \int_1^{a} \frac{1}{x^2}dx = \left[-\frac{1}{x}\right]_1^a = -\frac{1}{a} + \frac{1}{1} \xrightarrow[a \to \infty]{} 1$

Question 9:

a) i) The curve C crosses the $x - axis$ when $y = 0$

We solve $0 = \frac{(x+1)(x-3)}{x(x-2)}$ $\quad which\ gives\ (x+1)(x-3) = 0$

$$x = -1\ \ and\ \ x = 3$$

The curve crosses the x-axis at $(-1,0)$ and $(3,0)$

ii) • "$Vertical\ asymptote$" $x = 0\ and\ x = 2$ $\quad$ (roots of the denominator)

$$\bullet\ y = \frac{x^2 - 2x - 3}{x^2 - 2x} = \frac{1 - \frac{2}{x} - \frac{3}{x^2}}{1 - \frac{2}{x}} \xrightarrow[x \to \infty]{} 1 \quad so \quad y = 1\ is\ an\ asymptote.$$

b) The curve and the line y=k intersect if the equation $(y =)\frac{x^2 - 2x - 3}{x^2 - 2x} = k$ $has\ solutions.$

$$\frac{x^2 - 2x - 3}{x^2 - 2x} = k\ gives \quad x^2 - 2x - 3 = kx^2 - 2kx$$

$$(1-k)x^2 + (2k-2)x - 3 = 0$$

The discriminant : $\quad (2k-2)^2 - 4 \times (1-k) \times (-3)\ must\ be\ positive\ or = 0$

$$4k^2 - 8k + 4 + 12 - 12k \geq 0$$

$$4k^2 - 20k + 16 \geq 0$$

$$k^2 - 5k + 4 \geq 0$$

$$(k-1)(k-4) \geq 0$$

$ii)$ If the curve has only one stationary point, the line $y = k$ will intersect the curve at only one point, which means $(k-1)(k-4) = 0$

this gives two values for k : $k = 1$ or $k = 4$

$Case\ 1: k = 1$

then the equation $(1-k)x^2 + (2k-2)x - 3 = 0$ *becomes* $-3 = 0$

There is no solution

$Case\ 2: k = 4$

then the equation $(1-k)x^2 + (2k-2)x - 3 = 0$ *becomes* $-3x^2 + 6x - 3 = 0$

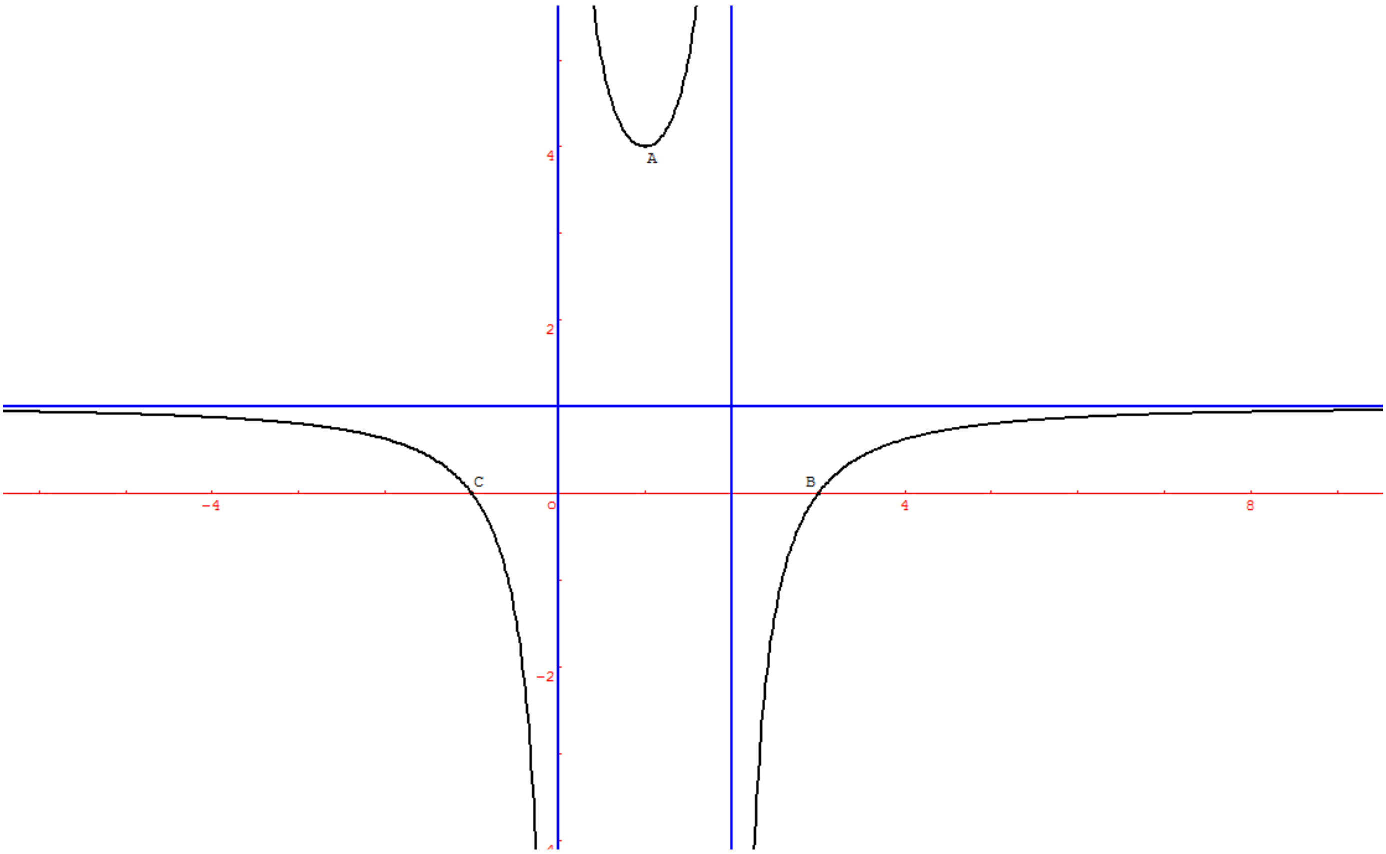
$$x^2 - 2x + 1 = 0$$

$$(x-1)^2 = 0$$

$$so\ \ x = 1\ \ and\ \ y = \frac{(1+1)(1-3)}{1(1-2)} = 4$$

The stationary point has coordinates $(1,4)$

$c)$



AQA June Examinations 2006

Scaled Mark Unit Grade Boundaries (GCE Specifications)

Unit Code	Unit Title	Maximum Scaled Mark	A	B	C	D	E
			Scaled Mark Grade Boundaries				
MFP1	GCE MATHEMATICS UNIT FP1	75	60	52	44	36	29

General Certificate of Education
January 2007
Advanced Subsidiary Examination

MATHEMATICS **MFP1**
Unit Further Pure 1

Friday 26 January 2007 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 (a) Solve the following equations, giving each root in the form $a+b\mathrm{i}$:

(i) $x^2+16=0$; *(2 marks)*

(ii) $x^2-2x+17=0$. *(2 marks)*

(b) (i) Expand $(1+x)^3$. *(2 marks)*

(ii) Express $(1+\mathrm{i})^3$ in the form $a+b\mathrm{i}$. *(2 marks)*

(iii) Hence, or otherwise, verify that $x=1+\mathrm{i}$ satisfies the equation

$$x^3+2x-4\mathrm{i}=0$$ *(2 marks)*

2 The matrices **A** and **B** are given by

$$\mathbf{A}=\begin{bmatrix}\frac{\sqrt{3}}{2} & -\frac{1}{2}\\ \frac{1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix},\quad \mathbf{B}=\begin{bmatrix}\frac{\sqrt{3}}{2} & \frac{1}{2}\\ \frac{1}{2} & -\frac{\sqrt{3}}{2}\end{bmatrix}$$

(a) Calculate:

(i) $\mathbf{A}+\mathbf{B}$; *(2 marks)*

(ii) $\mathbf{BA}$. *(3 marks)*

(b) Describe fully the geometrical transformation represented by each of the following matrices:

(i) $\mathbf{A}$; *(2 marks)*

(ii) $\mathbf{B}$; *(2 marks)*

(iii) $\mathbf{BA}$. *(2 marks)*

3 The quadratic equation

$$2x^2 + 4x + 3 = 0$$

has roots α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) Show that $\alpha^2 + \beta^2 = 1$. *(3 marks)*

(c) Find the value of $\alpha^4 + \beta^4$. *(3 marks)*

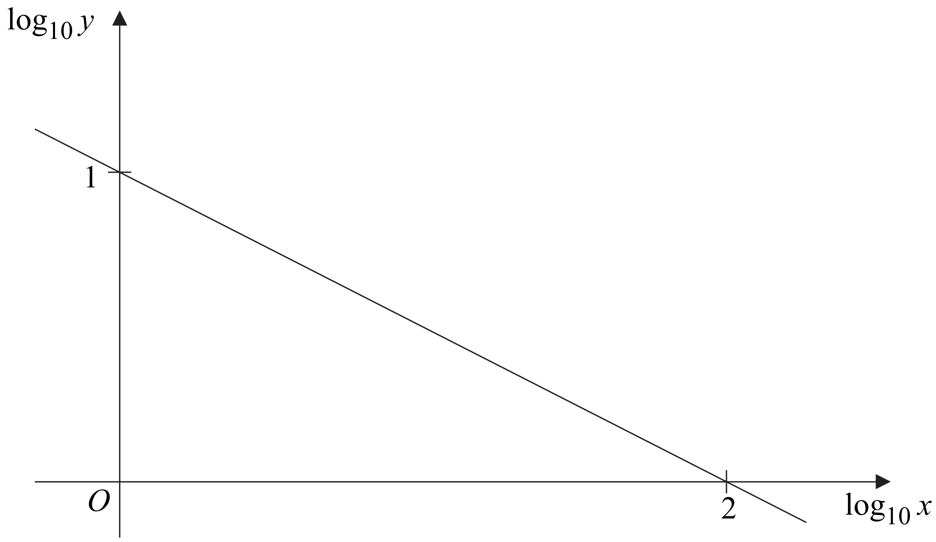
4 The variables x and y are related by an equation of the form

$$y = ax^b$$

where a and b are constants.

(a) Using logarithms to base 10, reduce the relation $y = ax^b$ to a linear law connecting $\log_{10} x$ and $\log_{10} y$. *(2 marks)*

(b) The diagram shows the linear graph that results from plotting $\log_{10} y$ against $\log_{10} x$.



Find the values of a and b. *(4 marks)*

5 A curve has equation

$$y = \frac{x}{x^2 - 1}$$

(a) Write down the equations of the three asymptotes to the curve. *(3 marks)*

(b) Sketch the curve.

(You are given that the curve has no stationary points.) *(4 marks)*

(c) Solve the inequality

$$\frac{x}{x^2 - 1} > 0$$ *(3 marks)*

6 (a) (i) Expand $(2r - 1)^2$. *(1 mark)*

(ii) Hence show that

$$\sum_{r=1}^{n} (2r - 1)^2 = \tfrac{1}{3} n(4n^2 - 1)$$ *(5 marks)*

(b) Hence find the sum of the squares of the odd numbers between 100 and 200. *(4 marks)*

7 The function f is defined for all real numbers by

$$\mathrm{f}(x) = \sin\left(x + \frac{\pi}{6}\right)$$

(a) Find the general solution of the equation $\mathrm{f}(x) = 0$. *(3 marks)*

(b) The quadratic function g is defined for all real numbers by

$$\mathrm{g}(x) = \frac{1}{2} + \frac{\sqrt{3}}{2}x - \frac{1}{4}x^2$$

It can be shown that $\mathrm{g}(x)$ gives a good approximation to $\mathrm{f}(x)$ for small values of x.

(i) Show that $\mathrm{g}(0.05)$ and $\mathrm{f}(0.05)$ are identical when rounded to four decimal places. *(2 marks)*

(ii) A chord joins the points on the curve $y = \mathrm{g}(x)$ for which $x = 0$ and $x = h$. Find an expression in terms of h for the gradient of this chord. *(2 marks)*

(iii) Using your answer to part (b)(ii), find the value of $\mathrm{g}'(0)$. *(1 mark)*

8 A curve C has equation

$$\frac{x^2}{25} - \frac{y^2}{9} = 1$$

(a) Find the y-coordinates of the points on C for which $x = 10$, giving each answer in the form $k\sqrt{3}$, where k is an integer. *(3 marks)*

(b) Sketch the curve C, indicating the coordinates of any points where the curve intersects the coordinate axes. *(3 marks)*

(c) Write down the equation of the tangent to C at the point where C intersects the positive x-axis. *(1 mark)*

(d) (i) Show that, if the line $y = x - 4$ intersects C, the x-coordinates of the points of intersection must satisfy the equation

$$16x^2 - 200x + 625 = 0$$

(3 marks)

(ii) Solve this equation and hence state the relationship between the line $y = x - 4$ and the curve C. *(2 marks)*

END OF QUESTIONS

MFP1

Q	Solution	Marks	Total	Comments
1(a)(i)	Roots are $\pm 4\mathrm{i}$	M1A1	2	M1 for one correct root or two correct factors
(ii)	Roots are $1 \pm 4\mathrm{i}$	M1A1	2	M1 for correct method
(b)(i)	$(1+x)^3 = 1 + 3x + 3x^2 + x^3$	M1A1	2	M1A0 if one small error
(ii)	$(1+\mathrm{i})^3 = 1 + 3\mathrm{i} - 3 - \mathrm{i} = -2 + 2\mathrm{i}$	M1A1	2	M1 if $\mathrm{i}^2 = -1$ used
(iii)	$(1+\mathrm{i})^3 + 2(1+\mathrm{i}) - 4\mathrm{i}$	M1		with attempt to evaluate
	$\ldots = (-2+2\mathrm{i}) + (2-2\mathrm{i}) = 0$	A1	2	convincingly shown (**AG**)
	Total		**10**	
2(a)(i)	$\mathbf{A}+\mathbf{B} = \begin{bmatrix} \sqrt{3} & 0 \\ 1 & 0 \end{bmatrix}$	M1A1	2	M1A0 if 3 entries correct; Condone $\frac{2\sqrt{3}}{2}$ for $\sqrt{3}$
(ii)	$\mathbf{BA} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$	B3,2,1	3	Deduct one for each error; **SC** B2,1 for **AB**
(b)(i)	Rotation 30° anticlockwise (abt O)	M1A1	2	M1 for rotation
(ii)	Reflection in $y = (\tan 15^\circ)x$	M1A1	2	M1 for reflection
(iii)	Reflection in x-axis	B2F	2	1/2 for reflection in y-axis ft (M1A1) only for the **SC**
	Alt: Answer to (i) followed by answer to (ii)	M1A1F	(2)	M1A0 if in wrong order or if order not made clear
	Total		**11**	
3(a)	$\alpha+\beta = -2,\ \alpha\beta = \frac{3}{2}$	B1B1	2	
(b)	Use of expansion of $(\alpha+\beta)^2$	M1		
	$\alpha^2+\beta^2 = (-2)^2 - 2\left(\frac{3}{2}\right) = 1$	m1A1	3	convincingly shown (**AG**); m1A0 if $\alpha+\beta = 2$ used
(c)	$\alpha^4+\beta^4$ given in terms of $\alpha+\beta, \alpha\beta$ and/or $\alpha^2+\beta^2$	M1A1		M1A0 if num error made
	$\alpha^4+\beta^4 = -\frac{7}{2}$	A1	3	OE
	Total		**8**	

MFP1 (cont)

Q	Solution	Marks	Total	Comments
4(a)	$\lg y = \lg a + b \lg x$	M1A1	2	M1 for use of one log law
(b)	Use of above result	M1		
	$a = 10$	A1		
	b = gradient	m1		OE; PI by answer $\pm\frac{1}{2}$
	$\ldots = -\frac{1}{2}$	A1	4	
	Total		**6**	
5(a)	Asymptotes $y = 0$, $x = -1$, $x = 1$	B1 × 3	3	
(b)	Three branches approaching two vertical asymptotes	B1		Asymptotes not necessarily drawn
	Middle branch passing through O	B1		with no stationary points
	Curve approaching $y = 0$ as $x \to \pm\infty$	B1		
	All correct	B1	4	with asymptotes shown and curve approaching all asymptotes correctly
(c)	Critical values $x = -1$, 0 and 1	B1		
	Solution set $-1 < x < 0$, $x > 1$	M1A1	3	M1 if one part correct or consistent with c's graph
	Total		**10**	
6(a)(i)	$(2r-1)^2 = 4r^2 - 4r + 1$	B1	1	
(ii)	$\sum(2r-1)^2 = 4\sum r^2 - 4\sum r + \sum 1$	M1		
	$\ldots = \frac{4}{3}n^3 - \frac{4}{3}n + \sum 1$	m1A1		
	$\sum 1 = n$	B1		
	Result convincingly shown	A1	5	**AG**
(b)	Sum = f (100) – f (50)	M1A1		M1 for 100 ± 1 and 50 ± 1
	$\ldots = 1\,166\,650$	A2	4	**SC** f(100) – f(51) = 1 156 449**:** 3/4
	Total		**10**	

MFP1 (cont)

Q	Solution	Marks	Total	Comments
7(a)	Particular solution, eg $-\frac{\pi}{6}$ or $\frac{5\pi}{6}$	B1		Degrees or decimals penalised in 3rd mark only
	Introduction of $n\pi$ or $2n\pi$	M1		
	GS $x=-\frac{\pi}{6}+n\pi$	A1F	3	OE(accept unsimplified); ft incorrect first solution
(b)(i)	f (0.05) ≈ 0.542 66 g (0.05) ≈ 0.542 68	B1 B1	2	either value AWRT 0.5427 both values correct to 4DP
(ii)	$\frac{g(h)-g(0)}{h}=\frac{\sqrt{3}}{2}-\frac{1}{4}h$	M1A1	2	M1A0 if num error made
(iii)	As $h\to 0$ this gives $g'(0)=\frac{\sqrt{3}}{2}$	A1F	1	AWRT 0.866; ft num error
	Total		**8**	
8(a)	$x=10\Rightarrow 4-\frac{y^2}{9}=1$	M1		
	$\Rightarrow y^2=27$	A1		PI
	$\Rightarrow y=\pm 3\sqrt{3}$	A1	3	
(b)	One branch generally correct Both branches correct Intersections at (± 5, 0)	B1 B1 B1	3	Asymptotes not needed With implied asymptotes
(c)	Required tangent is $x = 5$	B1F	1	ft wrong value in (b)
(d)(i)	y correctly eliminated Fractions correctly cleared $16x^2-200x+625=0$	M1 m1 A1	3	 convincingly shown (**AG**)
(ii)	$x=\frac{25}{4}$ Equal roots ⇒ tangency	B1 E1	2	No need to mention repeated root, but B0 if other values given as well Accept 'It's a tangent'
	Total		**12**	
	TOTAL		**75**	

Further pure 1 - AQA - January 2007

Question 1:

$a)i)\ x^2+16=0$

$x^2=-16 \qquad x=4i \ \ or \ \ x=-4i$

$ii)\ x^2-2x+17=0 \qquad \text{Discriminant: } (-2)^2-4\times1\times17=-64=(8i)^2$

$$The\ roots\ are\ \ x_1=\frac{2+8i}{2}=1+4i \quad or \quad x_2=1-4i$$

$b)i)\ (1+x)^3=1+3x+3x^2+x^3$

$ii)\ (1+i)^3=1+3i-3-i=-2+2i$

$iii)$ Let's work out $(1+i)^3+2(1+i)-4i$

$=-2+2i+2(1+i)-4i$

$=-2+2i+2+2i-4i=0$

$x=1+i \ \ is\ a\ solution\ \ to\ \ the\ \ equation\ \ x^3+2x-4i=0$

This opening question gave almost all the candidates the opportunity to score a high number of marks. Even when careless errors were made, for example the omission of the plus-or-minus symbol in one or both sections of part (a), there was much correct work for the examiners to reward. The expansion of the cube of a binomial expression in part (b)(i) seemed to be tackled more confidently than in the past. Almost all the candidates used $i^2=-1$ in part (b)(ii), though some were unsure how to deal with i^3.

Question 2:

$$a)i)\ A=\begin{bmatrix}\frac{\sqrt{3}}{2} & -\frac{1}{2}\\ \frac{1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}+B=\begin{bmatrix}\frac{\sqrt{3}}{2} & \frac{1}{2}\\ \frac{1}{2} & -\frac{\sqrt{3}}{2}\end{bmatrix}=\begin{bmatrix}\sqrt{3} & 0\\ 1 & 0\end{bmatrix}$$

$$ii)\ BA=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$$

$b)i)\ A=\begin{bmatrix}Cos30^o & -Sin30^o\\ Sin30^o & Cos30^o\end{bmatrix}$ A represents the rotation centre O, 30^o *anticlockwise*

$ii)\ B=\begin{bmatrix}Cos30^o & Sin30^o\\ Sin30^o & -Cos30^o\end{bmatrix}$ B represents the reflection in the line $y=(\tan 15^o)x$

$iii)\ BA=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$ BA represents the reflection in the line $y=0$ $(in\ the\ x-axis)$

Like Question 1, this question was very productive for the majority of candidates, who showed a good grasp of matrices and transformations. A strange error in part (a)(i) was a failure to simplify the expression $\frac{2\sqrt{3}}{2}$, though on this occasion the error was condoned. The most common mistake in part (a)(ii) was to multiply the two matrices the wrong way round. Only one mark was lost by this as long as the candidate made no other errors and was able to interpret the resulting product matrix as a transformation in part (b)(iii). Occasionally a candidate misinterpreted the 2θ occurring in the formula booklet for reflections, and gave the mirror line as *y*=*x* tan 60° instead of *y*=*x* tan15°.

Question 3:

$2x^2+4x+3=0 \quad has \ \ roots \ \ \alpha \ \ and \ \ \beta$

$$a)\ \alpha+\beta=\frac{-4}{2}=-2 \qquad and \qquad \alpha\beta=\frac{3}{2}$$

$$b)\ \alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=(-2)^2-2\times\frac{3}{2}=4-3=1$$

$$c)\ \alpha^4+\beta^4=(\alpha^2+\beta^2)^2-2\alpha^2\beta^2=1^2-2\left(\frac{3}{2}\right)^2=1-\frac{18}{4}=-\frac{7}{2}$$

It was pleasing to note that almost all candidates were aware that the sum of the roots was −2 and not +2, and that they were able to tackle the sum of the squares of the roots correctly in part (b). Part (c) was not so well answered. Relatively few candidates saw the short method based on the use of $(\alpha^2+\beta^2)^2$. Of those who used the expansion of $(\alpha+\beta)^4$, many found the correct expansion but still had difficulty arranging the terms so that the appropriate substitutions could be made.

Question 4:

$$a)\ y = ax^b \qquad so \qquad \log_{10} y = \log_{10}(ax^b)$$

$$\log_{10} y = \log_{10} a + b\log_{10} x$$

$$\log_{10} y = b\log_{10} x + \log_{10} a$$

$$Y = bX + c$$

$$b)\ When\ \log_{10} x = 0, \log_{10} y = 1 \qquad so\quad \log_{10} a = 1 \qquad a = 10$$

$$When\ \log_{10} x = 2, \log_{10} y = 0 \qquad so\quad 0 = 2b + 1 \qquad b = -\frac{1}{2}$$

Part (a) of this question was well answered, most candidates being familiar with the equation $y = ax^b$ and the technique needed to convert it into linear form. Some candidates seemed less happy with part (b), but most managed to show enough knowledge to score well here. Errors often arose from confusion between the intercept 1 on the vertical axis and the corresponding value of *y*, which required the taking of an antilogarithm.

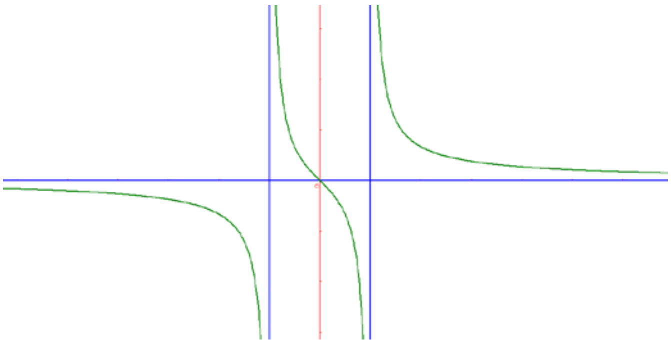
Question 5:

$$a)\ y = \frac{x}{x^2 - 1} = \frac{x}{(x-1)(x+1)}$$

$$"vertical\ asymptotes"\ x = 1\ \ and\ \ x = -1 \qquad \text{(roots of the denominator)}$$

$$y = \frac{\frac{1}{x^2}}{1 - \frac{1}{x^2}} \xrightarrow[x\to\infty]{} 0 \qquad y = 0\ is\ asymptote\ to\ the\ curve.$$

b)



$$c)\ \frac{x}{x^2 - 1} > 0$$

$$\frac{x}{x^2 - 1} = 0 \quad for \quad x = 0$$

By plotting the line $y = 0$ on the graph, the we conclude that

$$\frac{x}{x^2 - 1} > 0\ when\ -1 < x < 0\ or\ x > 1$$

Most candidates scored well here, picking up marks in all three parts of the question. Those who failed to obtain full marks in part (a) were usually candidates who gave *y* =1 instead of *y* = 0 as the equation of the horizontal asymptote. The sketch was usually reasonable but some candidates showed a stationary point, usually at or near the origin, despite the helpful information given in the question. There were many correct attempts at solving the inequality in part (c), though some answers bore no relation to the candidate's graph.

Question 6:

$$a)i)\ (2r-1)^2 = 4r^2-4r+1$$

$$ii)\sum_{r=1}^{n}(2r-1)^2 = \sum_{r=1}^{n}4r^2-4r+1 = 4\sum_{r=1}^{n}r^2-4\sum_{r=1}^{n}r+\sum_{r=1}^{n}1$$

$$= 4\times\frac{1}{6}n(n+1)(2n+1)-4\times\frac{1}{2}n(n+1)+n$$

$$= \frac{1}{3}n\left[2(n+1)(2n+1)-6(n+1)+3\right]$$

$$= \frac{1}{3}n\left[4n^2+6n+2-6n-6+3\right]$$

$$= \frac{1}{3}n(4n^2-1)$$

$$b)\ S = \sum_{r=1}^{100}(2r-1)^2-\sum_{r=1}^{50}(2r-1)^2 = \frac{1}{3}\times100\times(4\times100^2-1)-\frac{1}{3}\times50\times(4\times50^2-1)$$

$$= 1333300-166650 = 1166650$$

The simple request in part (a)(i) seemed to have the desired effect of setting the candidates along the right road in part (a)(ii). As usual many candidates struggled with the algebra but made reasonable progress. They could not hope to reach the printed answer legitimately if they equated Σ1 to 1 rather than to *n*. Very few, even among the strongest candidates, achieved anything worthwhile in part (b), most using *n* = 200 instead of *n* =100 at the top end. No credit was given for simply writing down the correct answer without any working, as the question required the candidates to use the formula previously established rather than summing the numbers directly on a calculator.

Question 7:

$$a)\ Sin\left(x+\frac{\pi}{6}\right)=0$$

$$x+\frac{\pi}{6}=0+k\pi$$

$$x=-\frac{\pi}{6}+k\pi \qquad k\in\mathbb{Z}$$

$$b)i)\ g(0.05)=\frac{1}{2}+\frac{\sqrt{3}}{2}\times0.05-\frac{1}{4}(0.05)^2$$

$$\approx 0.542676 = 0.5427 \text{ rounded to 4 decimal places}$$

$$f(0.05)\approx 0.5426583604 = 0.5427 \text{ rounded to 4 decimal places}$$

$$ii)\ \text{The gradient is } \frac{g(h)-g(0)}{h-0}=\frac{1}{h}\left(\frac{1}{2}+\frac{\sqrt{3}}{2}h-\frac{1}{4}h^2-\frac{1}{2}\right)$$

$$=\frac{1}{h}\left(\frac{\sqrt{3}}{2}h-\frac{1}{4}h^2\right)=\frac{\sqrt{3}}{2}-\frac{1}{4}h$$

$$iii)\ \text{When } h \text{ tends to 0, } \frac{g(h)-g(0)}{h-0}=\frac{\sqrt{3}}{2}-\frac{1}{4}h \text{ tends to } \frac{\sqrt{3}}{2}$$

$$so \qquad g'(0)=\frac{\sqrt{3}}{2}$$

The trigonometric equation in part (a) was more straightforward than usual and was correctly and concisely answered by a good number of candidates. Some earned two marks by finding a correct particular solution and introducing a term *n*π (or 2*n*π), but a common mistake was to use the formula nπ+(-1)^n α with α equated to the particular solution rather than to 0.

Part (b)(i) was usually answered adequately. The best candidates gave more than four decimal places, showing that the two numbers were different, before rounding them both to four decimal places. The examiners on this occasion tolerated a more casual approach. There was an encouraging response to the differentiation from first principles called for in parts (b)(ii) and (b)(iii). No credit could be given in part (b)(iii) to those who simply wrote down the answer, as by doing so they were not showing any knowledge of the required technique. Strictly speaking the candidates should have used the phrase 'as *h* tends to zero' or 'as *h* → 0', but the examiners allowed the mark for 'when *h* equals zero' or the equating of *h* to 0, even though zero is the one value of *h* for which the chord referred to in part (b)(ii) does not exist.

Question 8:

$$a)\ \frac{x^2}{25}-\frac{y^2}{9}=1 \qquad when\ \ x=10$$

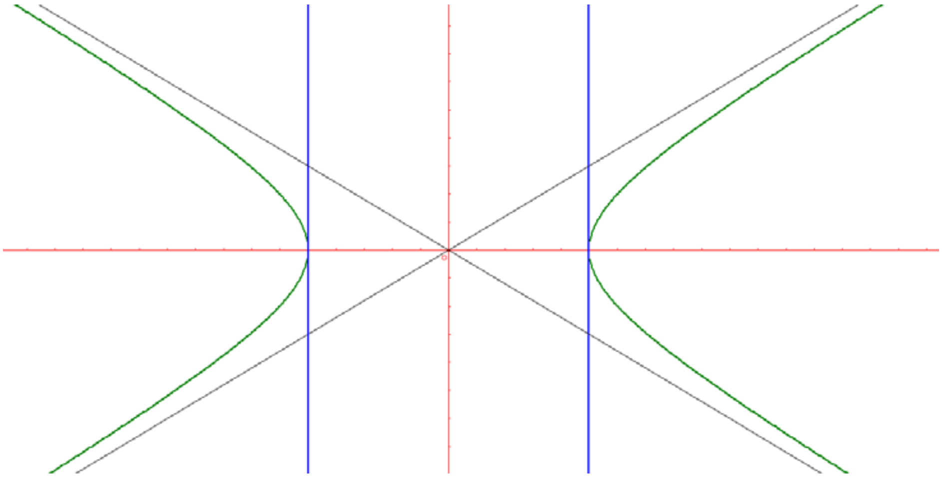
$$\frac{100}{25}-\frac{y^2}{9}=1 \qquad \frac{y^2}{9}=3 \qquad y^2=27$$

$$y=\pm\sqrt{27}$$

$$y=3\sqrt{3}\ \ or\ \ y=-3\sqrt{3}$$

$b)$ When $x=0,\ \ y=\pm 3$

When $y=0,\ \ x=\pm 5$



$c)$ C intersects the positive x-axis at the point $(5,0)$

so the tangent has equation $x=5$

$d)\,i)$ If $y=x-4$ intersects the curve,

then the x-coordinate of the point(s) of intersection

satisfies the following equation: $\frac{x^2}{25}-\frac{(x-4)^2}{9}=1 \qquad (\times 225)$

$$9x^2-25(x-4)^2=225$$

$$9x^2-25x^2+200x-400=225$$

$$-16x^2+200x-625=0$$

$$16x^2-200x+625=0$$

$ii)$ Discriminant: $(-200)^2-4\times 625\times 16=0$

$$x=\frac{200}{32}=\frac{25}{4} \quad and \quad y=\frac{9}{4}$$

This is a repeated root, so the line is TANGENT to the curve.

Part (a) of this question was generally well answered apart from the omission of the plus-or-minus symbol by a sizeable minority of candidates. The wording of the question should have made it very clear that there would be more than one point of intersection.

In part (b) many candidates sketched an ellipse instead of a hyperbola. Those who realised that it should be a hyperbola often lost a mark by showing too much curvature in the parts where the curve should be approaching its asymptotes.

Most candidates gave an appropriate answer (in the light of their graph) to part (c). No credit was given here to those who had drawn an incorrect curve, such as an ellipse, which just happened to have a vertical tangent at its intersection with the positive *x*-axis.

It was good to see that, no doubt helped by the printed answer, the majority of candidates
coped successfully with the clearing of fractions needed in part (d)(i). Some candidates
seemed to ignore the instruction to solve the equation in part (d)(ii) and went straight on to the
statement that the line must be a tangent to the curve. Others mentioned the equal roots of the
equation but failed to mention any relationship between the line and the curve.

Grade boundaries

Component Code	Component Title	Maximum Scaled Mark	A	B	C	D	E
			Scaled Mark Grade Boundaries				
MFP1	MATHEMATICS UNIT MFP1	75	60	53	46	39	32

General Certificate of Education
June 2007
Advanced Subsidiary Examination

MATHEMATICS **MFP1**
Unit Further Pure 1

Wednesday 20 June 2007 1.30 pm to 3.00 pm

For this paper you must have: • an 8-page answer book • the **blue** AQA booklet of formulae and statistical tables • an insert for use in Questions 5 and 9 (enclosed). You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- Fill in the boxes at the top of the insert.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The matrices **A** and **B** are given by

$$\mathbf{A} = \begin{bmatrix} 2 & 1 \\ 3 & 8 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

The matrix $\mathbf{M} = \mathbf{A} - 2\mathbf{B}$.

(a) Show that $\mathbf{M} = n\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$, where n is a positive integer. *(2 marks)*

(b) The matrix **M** represents a combination of an enlargement of scale factor p and a reflection in a line L. State the value of p and write down the equation of L. *(2 marks)*

(c) Show that

$$\mathbf{M}^2 = q\mathbf{I}$$

where q is an integer and **I** is the 2×2 identity matrix. *(2 marks)*

2 (a) Show that the equation

$$x^3 + x - 7 = 0$$

has a root between 1.6 and 1.8. *(3 marks)*

(b) Use interval bisection **twice**, starting with the interval in part (a), to give this root to one decimal place. *(4 marks)*

3 It is given that $z = x + \mathrm{i}y$, where x and y are real numbers.

(a) Find, in terms of x and y, the real and imaginary parts of

$$z - 3\mathrm{i}z^*$$

where z^* is the complex conjugate of z. *(3 marks)*

(b) Find the complex number z such that

$$z - 3\mathrm{i}z^* = 16$$ *(3 marks)*

4 The quadratic equation

$$2x^2 - x + 4 = 0$$

has roots α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) Show that $\dfrac{1}{\alpha} + \dfrac{1}{\beta} = \dfrac{1}{4}$. *(2 marks)*

(c) Find a quadratic equation with integer coefficients such that the roots of the equation are

$$\frac{4}{\alpha} \text{ and } \frac{4}{\beta}$$ *(3 marks)*

5 [**Figure 1** and **Figure 2**, printed on the insert, are provided for use in this question.]

The variables x and y are known to be related by an equation of the form

$$y = ab^x$$

where a and b are constants.

The following approximate values of x and y have been found.

x	1	2	3	4
y	3.84	6.14	9.82	15.7

(a) Complete the table in **Figure 1**, showing values of x and Y, where $Y = \log_{10} y$.
Give each value of Y to three decimal places. *(2 marks)*

(b) Show that, if $y = ab^x$, then x and Y must satisfy an equation of the form

$$Y = mx + c$$ *(3 marks)*

(c) Draw on **Figure 2** a linear graph relating x and Y. *(2 marks)*

(d) Hence find estimates for the values of a and b. *(4 marks)*

6 Find the general solution of the equation

$$\sin\left(2x - \frac{\pi}{2}\right) = \frac{\sqrt{3}}{2}$$

giving your answer in terms of π. *(6 marks)*

7 A curve has equation

$$y = \frac{3x - 1}{x + 2}$$

(a) Write down the equations of the two asymptotes to the curve. *(2 marks)*

(b) Sketch the curve, indicating the coordinates of the points where the curve intersects the coordinate axes. *(5 marks)*

(c) Hence, or otherwise, solve the inequality

$$0 < \frac{3x - 1}{x + 2} < 3$$ *(2 marks)*

8 For each of the following improper integrals, find the value of the integral **or** explain briefly why it does not have a value:

(a) $\displaystyle\int_0^1 \left(x^{\frac{1}{3}} + x^{-\frac{1}{3}}\right) \mathrm{d}x$; *(4 marks)*

(b) $\displaystyle\int_0^1 \frac{x^{\frac{1}{3}} + x^{-\frac{1}{3}}}{x} \mathrm{d}x$. *(4 marks)*

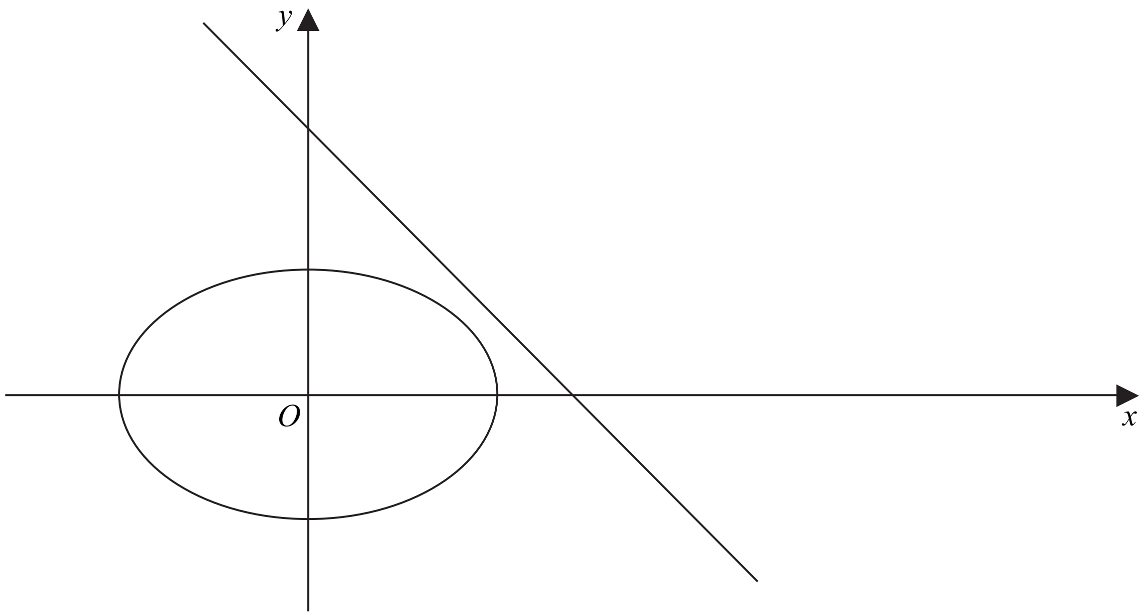
9 [**Figure 3**, printed on the insert, is provided for use in this question.]

The diagram shows the curve with equation

$$\frac{x^2}{2} + y^2 = 1$$

and the straight line with equation

$$x + y = 2$$



(a) Write down the exact coordinates of the points where the curve with equation $\frac{x^2}{2} + y^2 = 1$ intersects the coordinate axes. *(2 marks)*

(b) The curve is translated k units in the positive x direction, where k is a constant. Write down, in terms of k, the equation of the curve after this translation. *(2 marks)*

(c) Show that, if the line $x + y = 2$ intersects the **translated** curve, the x-coordinates of the points of intersection must satisfy the equation

$$3x^2 - 2(k+4)x + (k^2 + 6) = 0$$ *(4 marks)*

(d) Hence find the two values of k for which the line $x + y = 2$ is a tangent to the translated curve. Give your answer in the form $p \pm \sqrt{q}$, where p and q are integers. *(4 marks)*

(e) On **Figure 3**, show the translated curves corresponding to these two values of k. *(3 marks)*

END OF QUESTIONS

Surname		Other Names	
Centre Number		Candidate Number	
Candidate Signature			

General Certificate of Education
June 2007
Advanced Subsidiary Examination

MATHEMATICS **MFP1**
Unit Further Pure 1

ASSESSMENT and QUALIFICATIONS ALLIANCE

Insert

Insert for use in **Questions 5 and 9**.

Fill in the boxes at the top of this page.

Fasten this insert securely to your answer book.

Figure 1 (for use in Question 5)

x	1	2	3	4
Y	0.584			

Figure 2 (for use in Question 5)

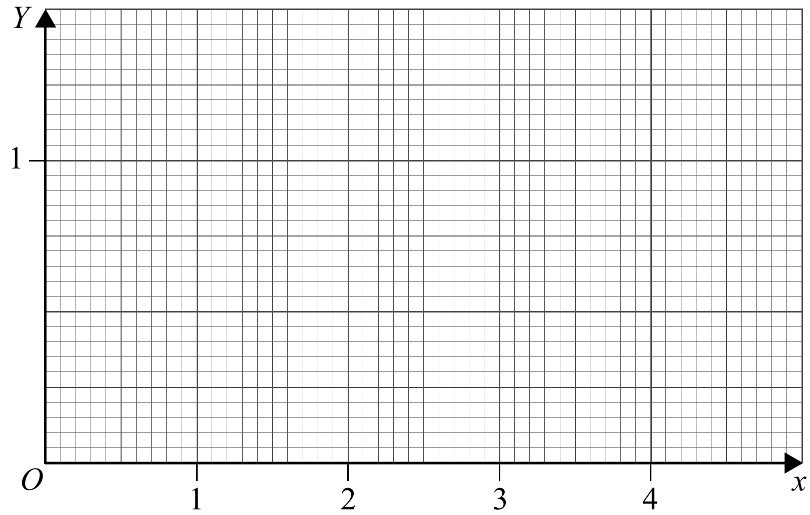
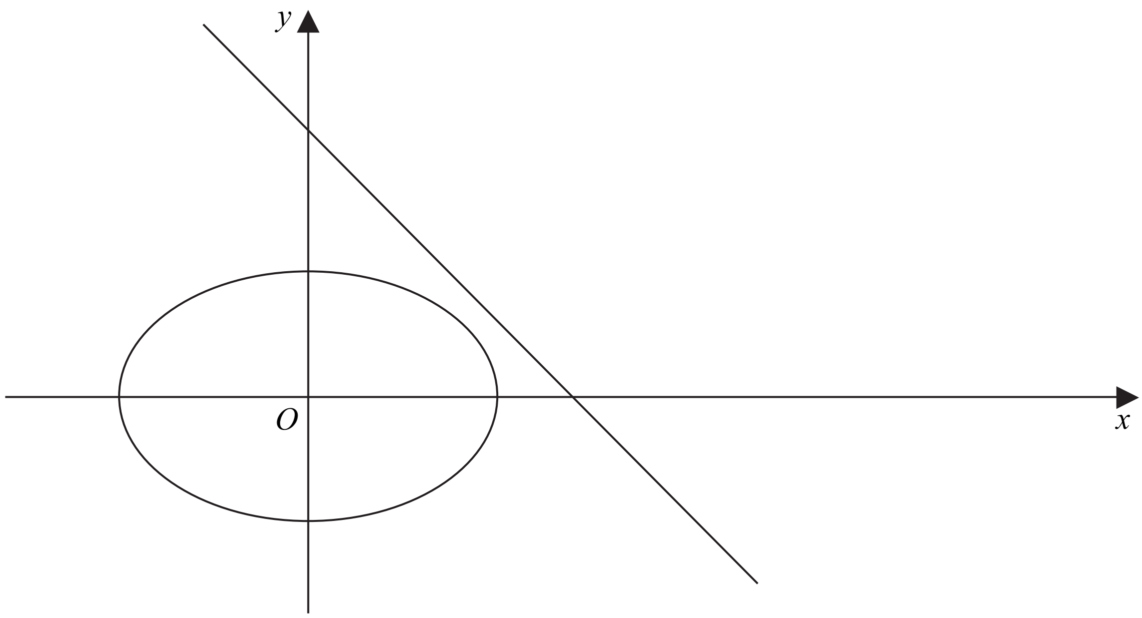


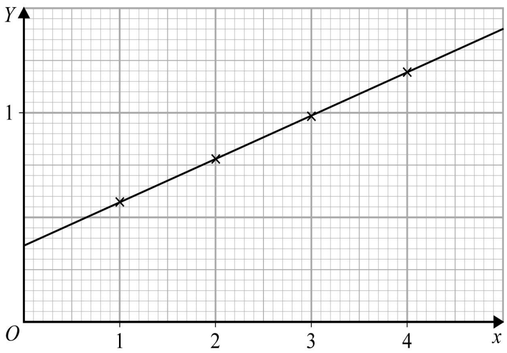
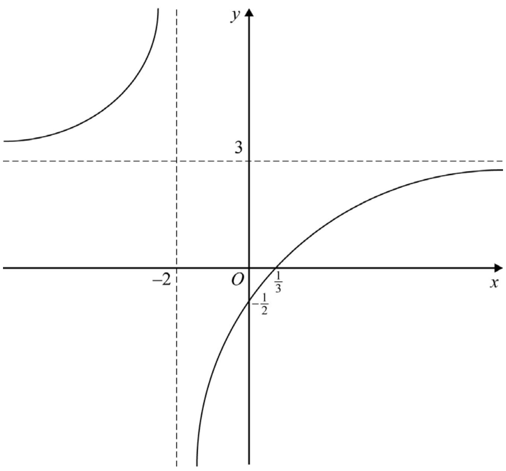
Figure 3 (for use in Question 9)



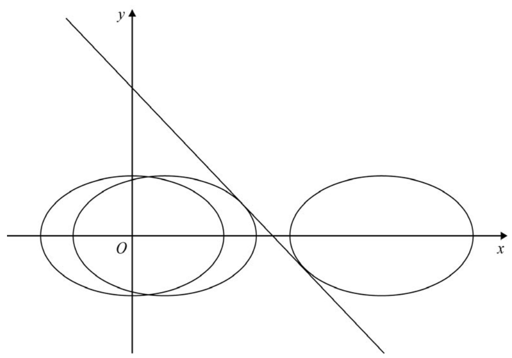
MFP1

Q	Solution	Mark	Total	Comments
1(a)	$\mathbf{M}=\begin{bmatrix}0 & -3\\ -3 & 0\end{bmatrix}$	B2,1	2	B1 if subtracted the wrong way round
(b)	$p = 3$	B1F		ft after B1 in (a)
	L is $y = -x$	B1	2	Allow $p = -3$, L is $y = x$
(c)	$\mathbf{M}^2=\begin{bmatrix}9 & 0\\ 0 & 9\end{bmatrix}$	B1F		Or by geometrical reasoning; ft as before
	$\ldots = 9\mathbf{I}$	B1F	2	ft as before
	Total		**6**	
2(a)	f(1.6) = −1.304, f(1.8) = 0.632	B1,B1		Allow 1 dp throughout
	Sign change, so root between	E1	3	
(b)	f(1.7) considered first	M1		
	f(1.7) = −0.387, so root > 1.7	A1		
	f(1.75) = 0.109375, so root ≈ 1.7	m1A1	4	m1 for f(1.65) after error
	Total		**7**	
3(a)	Use of $z^* = x - \mathrm{i}y$	M1		
	$z - 3\mathrm{i}z^* = x + \mathrm{i}y - 3\mathrm{i}x - 3y$	m1		Condone sign error here
	$\mathrm{R} = x - 3y$, $\mathrm{I} = -3x + y$	A1	3	Condone inclusion of i in I Allow if correct in (b)
(b)	$x - 3y = 16$, $-3x + y = 0$	M1		
	Elimination of x or y	m1		
	$z = -2 - 6\mathrm{i}$	A1F	3	Accept $x = -2$, $y = -6$; ft $x + 3y$ for $x - 3y$
	Total		**6**	
4(a)	$\alpha+\beta=\frac{1}{2},\ \alpha\beta=2$	B1B1	2	
(b)	$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}$	M1		
	$\ldots=\frac{\frac{1}{2}}{2}=\frac{1}{4}$	A1	2	Convincingly shown (AG)
(c)	Sum of roots = 1	B1F		PI by term $\pm x$; ft error(s) in (a)
	Product of roots = $\frac{16}{\alpha\beta}=8$	B1F		ft wrong value of $\alpha\beta$
	Equation is $x^2 - x + 8 = 0$	B1F	3	ft wrong sum/product; "= 0" needed
	Total		**7**	

MFP1 (cont)

Q	Solution	Mark	Total	Comments
5(a)	Values 0.788, 0.992, 1.196 in table	B2,1	2	B1 if one correct (or if wrong number of dp given)
(b)	lg $ab^x = \lg a + \lg b^x$	M1		
	$\lg b^x = x \lg b$	M1		
	So $Y = (\lg b)\, x + \lg a$	A1	3	Allow NMS
(c)		B1F		Four points plotted; ft wrong values in (a)
		B1F	2	Good straight line drawn; ft incorrect points
(d)	a = antilog of y-intercept	M1A1		Accept 2.23 to 2.52
	b = antilog of gradient	M1A1	4	Accept 1.58 to 1.62
	Total		**11**	
6	One value of $2x - \frac{\pi}{2}$ is $\frac{\pi}{3}$	B1		OE (PI); degrees/decimals penalised in 6th mark only
	Another value is $\pi - \frac{\pi}{3} = \frac{2\pi}{3}$	B1F		OE (PI); ft wrong first value
	Introduction of $2n\pi$ or $n\pi$	M1		
	General solution for x	m1		
	GS $x = \frac{5\pi}{12} + n\pi$ or $x = \frac{7\pi}{12} + n\pi$	A2,1	6	OE; A1 if one part correct
	Total		**6**	
7(a)	Asymptotes $x = -2$, $y = 3$	B1,B1	2	
(b)		B1		Curve approaching asymptotes
		B1,B1		Passing through $\left(\frac{1}{3}, 0\right)$ and $\left(0, -\frac{1}{2}\right)$
		B1,B1	5	Both branches generally correct B1 if two branches shown
(c)	Solution set is $x > \frac{1}{3}$	B2,1F	2	B1 for good attempt; ft wrong point of intersection
	Total		**9**	

MFP1 (cont)

Q	Solution	Marks	Totals	Comments
8(a)	$\int\left(x^{\frac{1}{3}}+x^{-\frac{1}{3}}\right)\mathrm{d}x=\frac{3}{4}x^{\frac{4}{3}}+\frac{3}{2}x^{\frac{2}{3}}\ (+c)$	M1A1		M1 for adding 1 to index at least once
	$\int_0^1 \ldots=\left(\frac{3}{4}+\frac{3}{2}\right)-0=\frac{9}{4}$	m1A1	4	Condone no mention of limiting process; m1 if "$-$ 0" stated or implied
(b)	Second term is $x^{-\frac{4}{3}}$	B1		
	Integral of this is $-3x^{-\frac{1}{3}}$	M1A1		M1 for correct index
	$x^{-\frac{1}{3}}\to\infty$ as $x\to0$, so no value	E1	4	
	Total		**8**	
9(a)	Intersections $\left(\pm\sqrt{2},0\right)$, (0, ± 1)	B1B1	2	Allow B1 for $\left(\sqrt{2},0\right)$, (0, 1)
(b)	Equation is $\frac{(x-k)^2}{2}+y^2=1$	M1A1	2	M1 if only one small error, eg $x+k$ for $x-k$
(c)	Correct elimination of y Correct expansion of squares Correct removal of denominator Answer convincingly established	M1 M1 M1 A1	4	AG
(d)	Tgt $\Rightarrow 4(k+4)^2-12(k^2+6)=0$ ... $\Rightarrow k^2-4k+1=0$ ... $\Rightarrow k=2\pm\sqrt{3}$	M1 m1A1 A1	4	OE
(e)		B1 B2	3	Curve to left of line Curve to right of line Curves must touch the line in approx correct positions SC 1/3 if both curves are incomplete but touch the line correctly
	Total		**15**	
	TOTAL		**75**	

Further pure 1 - AQA - June 2007

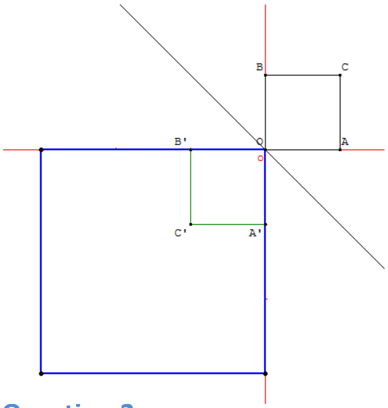
Question 1:

$$a)\ M = A - 2B = \begin{bmatrix} 2 & 1 \\ 3 & 8 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix} = \begin{bmatrix} 0 & -3 \\ -3 & 0 \end{bmatrix} = 3\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \qquad n = 3$$

$b)$ The matrix $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ is the reflexion in the line $y = -x$

The scale factor of the enlargement is $p = 3$

$$c)\ M^2 = 9\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 9I \qquad q = 9$$



This question was generally well answered. The most common errors were in part (b), where the mirror line was given as $y = x$ rather than $y = -x$. It was, however, acceptable to give this reflection in conjunction with an enlargement with scale factor −3.
In part (c) some candidates omitted the factor 3 before carrying out the matrix multiplication, while others multiplied incorrectly to obtain the 9s and 0s in all the wrong places.

Question 2:

$a)\ x^3 + x - 7 = 0 \qquad Let's\ call\ \ f(x) = x^3 + x - 7$

$$f(1.6) = -1.304 < 0$$

$$f(1.8) = 0.632 > 0$$

According to the "sign change" rule, we can say that there is at least one solution of the equation $f(x) = 0$ between 1.6 and 1.8

$b)\ Let's\ work\ out\ \ f(1.7)$

$$f(1.7) = -0.387 < 0$$

the solution is between 1.7 and 1.8

$Let's\ \ work\ out\ f(1.75)$

$$f(1.75) = 0.109375 > 0$$

The solution is between 1.70 and 1.75

This is 1.7 rounded to 1 decimal place

Almost all the candidates were able to make a good start to this question, though some were unable to draw a proper conclusion in part (a). In part (b) the response was again very good.
The most common error was not understanding what exactly was being asked for at the end of the question. If it is known that the root lies between 1.7 and 1.75, then it must be 1.7 to one decimal place. But many candidates gave no value to one decimal place, or gave a value of the function (usually 0.1) rather than a value of *x*.

Question 3:

$z = x + iy$

$a)\ z - 3iz^* = x + iy - 3i(x - iy) = x + iy - 3ix - 3y$

$$\mathrm{Re}(z - 3iz^*) = x - 3y$$

$$\mathrm{Im}(z - 3iz^*) = -3x + y$$

$$b)\ z - 3iz^* = 16 \qquad means \qquad \begin{cases} x - 3y = 16 \\ -3x + y = 0 \end{cases} \qquad \begin{cases} x - 3y = 16 \\ -9x + 3y = 0 \end{cases}$$

$This\ gives\ \ -8x = 16 \quad x = -2 \quad and \quad y = -6$

The solution is $z = -2 - 6i$

Although most of the candidates who took this paper are good at algebra, there are still quite a number who made elementary sign errors. The fourth term of the expansion in part (a) frequently came out with a plus instead of a minus. The error of including an 'i' in the imaginary part was condoned.
In part (b) some candidates seemed not to realise that they needed to equate real and imaginary parts, despite the hint in part (a). Others used 16 for both the real and imaginary parts of the right-hand side of the equation. But a large number solved correctly to obtain full marks. Those who had made the sign error already referred to were given full marks in part (b) if their work was otherwise faultless.

Question 4:

$2x^2 - x + 4 = 0$ *has roots* α *and* β

$a)\ \alpha + \beta = \frac{1}{2}$ and $\alpha\beta = \frac{4}{2} = 2$

$$b)\ \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta} = \frac{\frac{1}{2}}{2} = \frac{1}{4}$$

$c)$ Let's call $u = \frac{4}{\alpha}$ and $v = \frac{4}{\beta}$

we have $u + v = \frac{4}{\alpha} + \frac{4}{\beta} = 4\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) = 4 \times \frac{1}{4} = 1$

$$u\,v = \frac{4}{\alpha} \times \frac{4}{\beta} = \frac{16}{\alpha\beta} = \frac{16}{2} = 8$$

An equation with roots $u = \frac{4}{\alpha}$ and $v = \frac{4}{\beta}$ is $x^2 - 1x + 8 = 0$

$$x^2 - x + 8 = 0$$

Most candidates answered this question very well. Only a small number of candidates gave the wrong sign for the sum of the roots at the beginning of the question. Rather more made the equivalent mistake at the end of the question, giving the *x* term with the wrong sign. Another common mistake at the end of the question was to omit the 'equals zero', so that the equation asked for was not given as an equation at all.

Question 5:

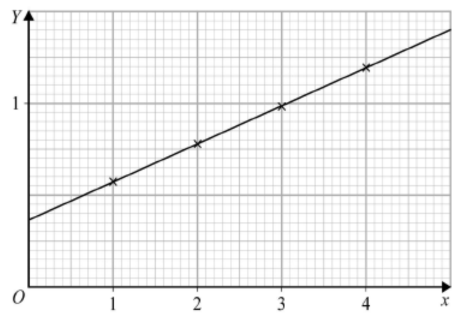
x	1	2	3	4
Y	0.584	0.788	0.992	1.196

$b)\ y = ab^x$ *so* $\log_{10} y = \log_{10}(ab^x)$

$$\log_{10} y = \log_{10} a + x \log_{10} b$$

$Y = mx + c$ *with* $m = \log_{10} b$ *and* $c = \log_{10} a$

c)



$d)$ *When* $x = 0,\ Y = c = \log_{10} a = 0.35$

$$a = 10^{0.35} \approx 2.24$$

$$\log_{10} b = gradient = \frac{1.196 - 0.584}{4 - 1} = 0.204$$

$$b = 10^{0.204} \approx 1.6$$

The great majority of candidates gave the correct values to three decimal places in part (a).
Most of them coped efficiently with the logarithmic manipulation in part (b), but occasionally a candidate would treat the expression ab^x as if it were $(ab)^x$, resulting in a loss of at least four marks, as it was now impossible to distinguish validly between *a* and *b*.
The plotting of the points on the graph was usually well done, but some candidates misread the vertical scale here, and again in part (d) when they attempted to read off the intercept on the *Y*-axis. Some candidates failed to use this intercept, resorting instead to more indirect methods of finding a value for log *a*. Methods for finding the gradient of the linear graph were equally clumsy. Some candidates, even after obtaining a correct equation in part (b), did not realise that the taking of antilogs was needed in part (d).

Question 6:

$$Sin\left(2x - \frac{\pi}{2}\right) = \frac{\sqrt{3}}{2} = Sin\left(\frac{\pi}{3}\right)$$

so $2x - \frac{\pi}{2} = \frac{\pi}{3} + k2\pi \qquad 2x = \frac{5\pi}{6} + k2\pi \qquad x = \frac{5\pi}{12} + k\pi$

or $2x - \frac{\pi}{2} = \pi - \frac{\pi}{3} + k2\pi \qquad 2x = \frac{7}{6}\pi + k2\pi \qquad x = \frac{7}{12}\pi + k\pi \quad k \in \mathbb{Z}$

As in past MFP1 papers, the question on trigonometric equations was not as well answered as most of the other questions. The use of radians presented a difficulty to many candidates, who seemed to have met the idea but not to have become really familiar with it. Most candidates knew that for a general solution it was necessary to introduce a term $2n\pi$ or something similar somewhere in the solution. Many, however, brought in this term at an inappropriate stage. Some had learnt by heart a formula for the general solution of the equation sin *x* = sin *α* , but applied it incorrectly. Many candidates earned 4 marks out of 6 for working correctly from one particular solution to the corresponding general solution, but omitting the other particular solution or finding it incorrectly.

Question 7:

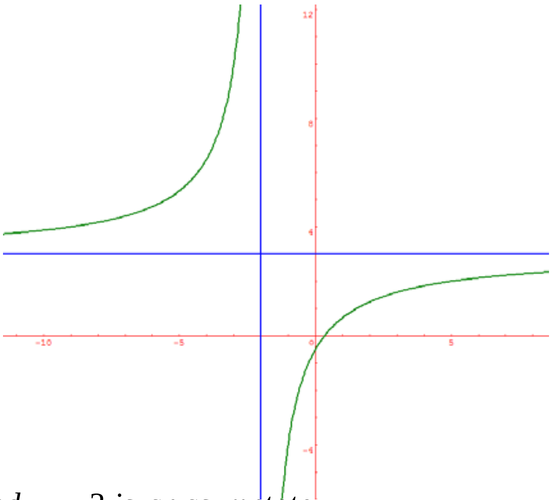
$$y=\frac{3x-1}{x+2}$$

$a)$ "vertical asymptote" $x=-2$

$$y=\frac{3-\frac{1}{x}}{1+\frac{2}{x}}\xrightarrow[x\to\infty]{}3 \qquad y=3 \text{ is asymptote to the curve}$$

$b)$ When $x=0,\ y=-\frac{1}{2}$

$y=0$ when $x=\frac{1}{3}$



$c)\ \frac{3x-1}{x+2}=0$ for $x=\frac{1}{3}$ and $y=3$ is an asymptote.

So, $0<\frac{3x-1}{x+2}<3$ for $x>\frac{1}{3}$

Many candidates scored well on parts (a) and (b) of this question, but relatively few candidates made a good attempt at part (c). In part (a) the asymptotes were usually found correctly, as were the coordinates of the required points of intersection in part (b). The graph in part (b) was usually recognisable as a hyperbola, or at least as one branch of a hyperbola, the other branch not being seen. In part (c) many candidates resorted to algebraic methods for solving inequalities rather than simply reading off the solution set from the graph. These algebraic methods, more often than not, were spurious.

Question 8:

$$a)\int x^{\frac{1}{3}}+x^{-\frac{1}{3}}dx=\frac{3}{4}x^{\frac{4}{3}}+\frac{3}{2}x^{\frac{2}{3}}+c$$

This function is defined between 0 and 1

$$\int_0^1 x^{\frac{1}{3}}+x^{-\frac{1}{3}}dx=\left[\frac{3}{4}x^{\frac{4}{3}}+\frac{3}{2}x^{\frac{2}{3}}\right]_0^1=\frac{3}{4}+\frac{3}{2}-0=\frac{9}{4}$$

$$b)\int \frac{x^{\frac{1}{3}}+x^{-\frac{1}{3}}}{x}dx=\int x^{-\frac{2}{3}}+x^{-\frac{4}{3}}dx=3x^{\frac{1}{3}}-3x^{-\frac{1}{3}}+c$$

$x^{-\frac{1}{3}}$ is not defined for $x=0$. no value

This question proved to be very largely a test of integration. Many candidates answered part (a) without any apparent awareness of a limiting process, but full marks were awarded if the answer was correct. The positive indices meant that the powers of *x* would tend to zero as *x* itself tended to zero. In part (b) a slightly more difficult integration led to one term having a negative index. Three marks were awarded for the integration, but for the final mark it was necessary to give some indication as to which term tended to infinity. Many candidates did not gain this last mark, but it was sad to see how many did not gain any marks at all in part (b). This was usually for one of two reasons. Either they failed to simplify the integrand and carried out a totally invalid process of integration, or they saw that the denominator *x* of the integrand would become zero and said that this made the integral 'improper' and therefore incapable of having any value. Since it was stated in the question that **both** integrals were improper, this comment failed to attract any sympathy from the examiners.

Question 9:

$$Ellipse: \frac{x^2}{2} + y^2 = 1 \quad and\ straight\ line\ x + y = 2$$

$a)$ $When\ x = 0,\quad y^2 = 1 \qquad y = \pm 1$

$\quad when\ y = 0,\quad x^2 = 2 \qquad x = \pm\sqrt{2}$

The curve intersects the axes at (0,1), (0,-1), $(-\sqrt{2},0)$, $(\sqrt{2},0)$

$$b)\ \frac{(x-k)^2}{2} + y^2 = 1$$

$c)$ $x + y = 2\ \ so\ \ y = 2 - x.$

The x-coordinate of the point(s) of intersection satisfy:

$$\frac{(x-k)^2}{2} + (2-x)^2 = 1$$

$$(x-k)^2 + 2(2-x)^2 = 2$$

$$x^2 - 2xk + k^2 + 8 - 8x + 2x^2 = 2$$

$$3x^2 - 2(k+4)x + k^2 + 6 = 0$$

$d)$ The line is tangent to the curve when this equation has a repeated root, meaning the discriminant is 0

$$\left(-2k-8\right)^2 - 4\times 3\times (k^2+6) = 0$$

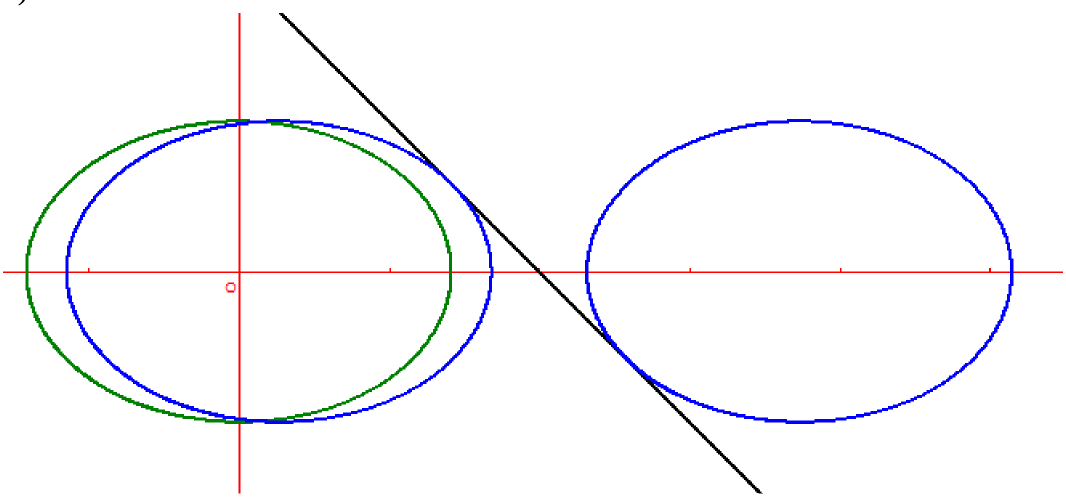
$$4k^2 + 32k + 64 - 12k^2 - 72 = 0$$

$$-8k^2 + 32k - 8 = 0$$

$$k^2 - 4k + 1 = 0$$

Using the quadratic formula, we have $k = 2 \pm \sqrt{3}$

$e)$



This final question presented a varied challenge, mostly focused on the algebraic skills needed to cope with the quadratic equation printed in part (c) of the question – deriving this equation from two other equations and then using the discriminant of the equation to find the cases where the ellipse would touch the line. Many candidates were well prepared for this type of question and scored heavily, though with occasional errors and omissions.

Part (e) of the question was often not attempted at all, or the attempts were totally incorrect, involving transformations other than translations parallel to the *x*-axis. In some cases only one of the two possible cases was illustrated, usually the one where the ellipse touched the straight line on the lower left side of the line, but an impressive minority of scripts ended with an accurate portrayal of both cases.

Grade boundaries

Component Code	Component Title	Maximum Scaled Mark	A	B	C	D	E
MFP1	GCE MATHEMATICS UNIT FP1	75	64	56	48	41	34

General Certificate of Education
January 2008
Advanced Subsidiary Examination

MATHEMATICS **MFP1**
Unit Further Pure 1

Friday 25 January 2008 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables
- an insert for use in Question 7 (enclosed).

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- Fill in the boxes at the top of the insert.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 It is given that $z_1 = 2 + \mathrm{i}$ and that $z_1{}^*$ is the complex conjugate of z_1.

Find the real numbers x and y such that

$$x + 3\mathrm{i}y = z_1 + 4\mathrm{i}z_1{}^*$$ *(4 marks)*

2 A curve satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 2^x$$

Starting at the point $(1, 4)$ on the curve, use a step-by-step method with a step length of 0.01 to estimate the value of y at $x = 1.02$. Give your answer to six significant figures. *(5 marks)*

3 Find the general solution of the equation

$$\tan 4\left(x - \frac{\pi}{8}\right) = 1$$

giving your answer in terms of π. *(5 marks)*

4 (a) Find

$$\sum_{r=1}^{n} (r^3 - 6r)$$

expressing your answer in the form

$$kn(n+1)(n+p)(n+q)$$

where k is a fraction and p and q are integers. *(5 marks)*

(b) It is given that

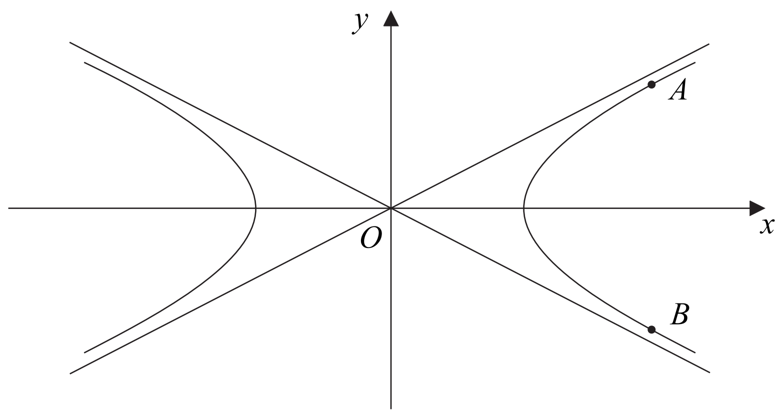
$$S = \sum_{r=1}^{1000} (r^3 - 6r)$$

Without calculating the value of S, show that S is a multiple of 2008. *(2 marks)*

5 The diagram shows the hyperbola

$$\frac{x^2}{4} - y^2 = 1$$

and its asymptotes.



(a) Write down the equations of the two asymptotes. *(2 marks)*

(b) The points on the hyperbola for which $x = 4$ are denoted by A and B.

Find, in surd form, the y-coordinates of A and B. *(2 marks)*

(c) The hyperbola and its asymptotes are translated by two units in the positive y direction.

Write down:

(i) the y-coordinates of the image points of A and B under this translation; *(1 mark)*

(ii) the equations of the hyperbola and the asymptotes after the translation. *(3 marks)*

6 The matrix $\mathbf{M}$ is defined by

$$\mathbf{M} = \begin{bmatrix} \sqrt{3} & 3 \\ 3 & -\sqrt{3} \end{bmatrix}$$

(a) (i) Show that

$$\mathbf{M}^2 = p\mathbf{I}$$

where p is an integer and $\mathbf{I}$ is the 2×2 identity matrix. *(3 marks)*

(ii) Show that the matrix $\mathbf{M}$ can be written in the form

$$q\begin{bmatrix} \cos 60^\circ & \sin 60^\circ \\ \sin 60^\circ & -\cos 60^\circ \end{bmatrix}$$

where q is a real number. Give the value of q in surd form. *(3 marks)*

(b) The matrix $\mathbf{M}$ represents a combination of an enlargement and a reflection.

Find:

(i) the scale factor of the enlargement; *(1 mark)*

(ii) the equation of the mirror line of the reflection. *(1 mark)*

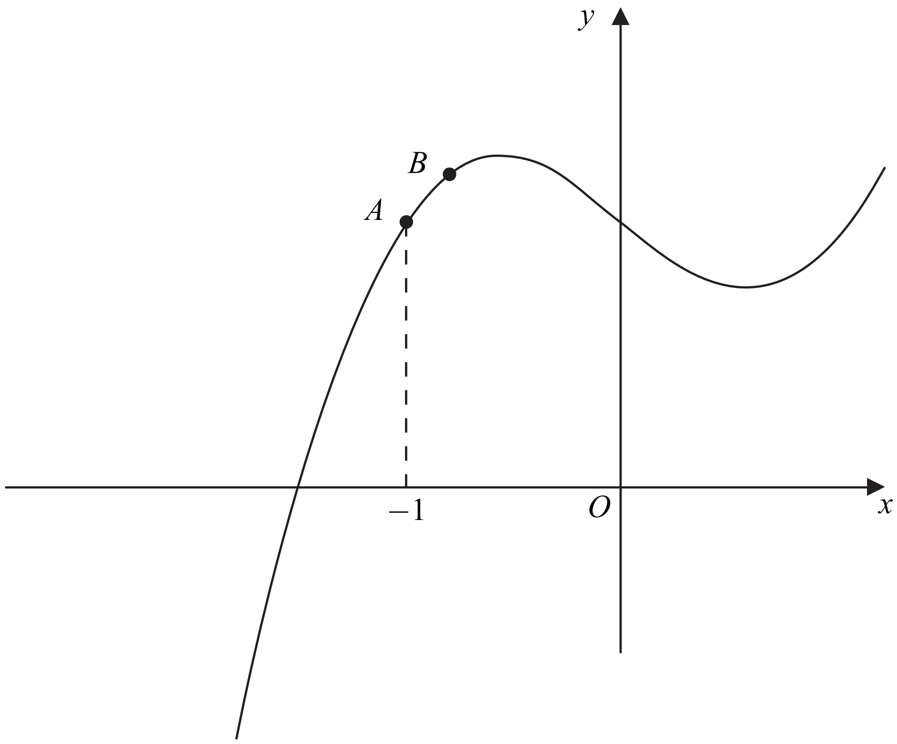
(c) Describe fully the geometrical transformation represented by $\mathbf{M}^4$. *(2 marks)*

7 [**Figure 1**, printed on the insert, is provided for use in this question.]

The diagram shows the curve

$$y = x^3 - x + 1$$

The points A and B on the curve have x-coordinates -1 and $-1 + h$ respectively.



(a) (i) Show that the y-coordinate of the point B is

$$1 + 2h - 3h^2 + h^3$$ *(3 marks)*

(ii) Find the gradient of the chord AB in the form

$$p + qh + rh^2$$

where p, q and r are integers. *(3 marks)*

(iii) Explain how your answer to part (a)(ii) can be used to find the gradient of the tangent to the curve at A. State the value of this gradient. *(2 marks)*

(b) The equation $x^3 - x + 1 = 0$ has one real root, α.

(i) Taking $x_1 = -1$ as a first approximation to α, use the Newton-Raphson method to find a second approximation, x_2, to α. *(2 marks)*

(ii) On **Figure 1**, draw a straight line to illustrate the Newton-Raphson method as used in part (b)(i). Show the points $(x_2, 0)$ and $(\alpha, 0)$ on your diagram. *(2 marks)*

8 (a) (i) It is given that α and β are the roots of the equation

$$x^2 - 2x + 4 = 0$$

Without solving this equation, show that α^3 and β^3 are the roots of the equation

$$x^2 + 16x + 64 = 0$$ *(6 marks)*

(ii) State, giving a reason, whether the roots of the equation

$$x^2 + 16x + 64 = 0$$

are real and equal, real and distinct, or non-real. *(2 marks)*

(b) Solve the equation

$$x^2 - 2x + 4 = 0$$ *(2 marks)*

(c) Use your answers to parts (a) and (b) to show that

$$(1 + \mathrm{i}\sqrt{3})^3 = (1 - \mathrm{i}\sqrt{3})^3$$ *(2 marks)*

9 A curve C has equation

$$y = \frac{2}{x(x-4)}$$

(a) Write down the equations of the three asymptotes of C. *(3 marks)*

(b) The curve C has one stationary point. By considering an appropriate quadratic equation, find the coordinates of this stationary point.

(No credit will be given for solutions based on differentiation.) *(6 marks)*

(c) Sketch the curve C. *(3 marks)*

END OF QUESTIONS

Surname		Other Names							
Centre Number						Candidate Number			
Candidate Signature									

General Certificate of Education
January 2008
Advanced Subsidiary Examination

MATHEMATICS **MFP1**
Unit Further Pure 1

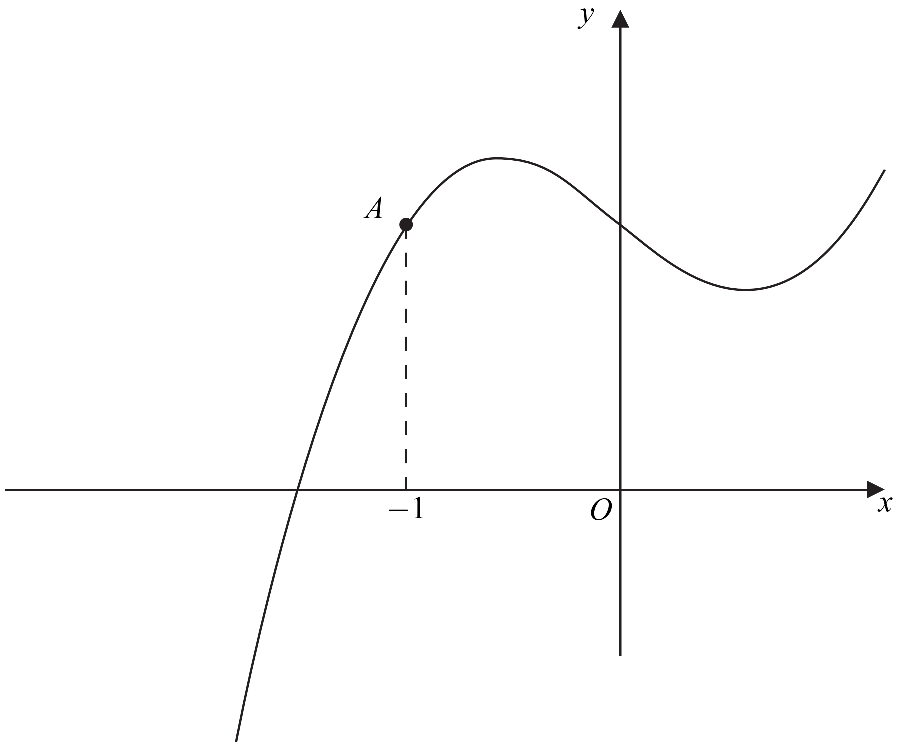
Insert

Insert for use in **Question 7**.

Fill in the boxes at the top of this page.

Fasten this insert securely to your answer book.

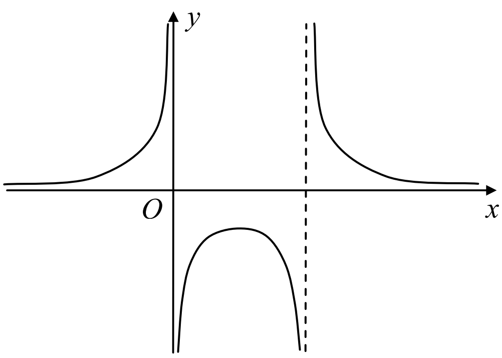
Figure 1 (for use in Question 7)



MFP1

Q	Solution	Marks	Totals	Comments
1	$z_1 + 4\mathrm{i}\, z_1^* = (2 + \mathrm{i}) + 4\mathrm{i}\,(2 - \mathrm{i})$	M1		Use of conjugate
	$\ldots = (2 + \mathrm{i}) + (8\mathrm{i} + 4)$	M1		Use of $\mathrm{i}^2 = -1$
	$\ldots = 6 + 9\mathrm{i}$, so $x = 6$ and $y = 3$	M1A1	4	M1 for equating Real and imaginary parts
	Total		**4**	
2	$0.01(2^1)$ added to value of y	M1		Variations possible here
	So $y(1.01) \approx 4.02$	A1		PI
	Second increment is $0.01(2^{1.01})$	m1		
	$\ldots \approx 0.020\ 139$	A1		
	So $y(1.02) \approx 4.040\ 14$	A1	5	
	Total		**5**	
3	Use of $\tan\frac{\pi}{4} = 1$	B1		Degrees or decimals penalised in last mark only
	Introduction of $n\pi$	M1		or kn at any stage
	Division of all terms by 4	m1		
	Addition of $\pi/8$	m1		OE
	GS $x = \frac{3\pi}{16} + \frac{n\pi}{4}$	A1	5	OE
	Total		**5**	
4(a)	Use of formula for $\sum r^3$ or $\sum r$	M1		
	n is a factor of the expression	m1		clearly shown
	So is $(n + 1)$	m1		ditto
	$S_n = \frac{1}{4}n(n+1)(n^2+n-12)$	A1		
	$\ldots = \frac{1}{4}n(n+1)(n+4)(n-3)$	A1F	5	ft wrong value for k
(b)	$n = 1000$ substituted into expression	m1		The factor 1004, or 1000 + 4, seen
	Conclusion convincingly shown	A1	2	not '2008 × 124749625'
	Need $\frac{1000}{4}$ is even, hence conclusion			OE
	Total		**7**	
5(a)	Asymptotes are $y = \pm\frac{1}{2}x$	M1A1	2	OE; M1 for $y = \pm\, mx$
(b)	$x = 4$ substituted into equation	M1		
	$y^2 = 3$ so $y = \pm\sqrt{3}$	A1	2	Allow NMS
(c)(i)	y-coords are $2 \pm \sqrt{3}$	B1F	1	ft wrong answer to (b)
(ii)	Hyperbola is $\frac{x^2}{4} - (y-2)^2 = 1$	M1A1		M1A0 if $y + 2$ used
	Asymptotes are $y = 2 \pm \frac{1}{2}x$	B1F	3	ft wrong gradients in (a)
	Total		**8**	
6(a)(i)	$\mathbf{M}^2 = \begin{bmatrix} 12 & 0 \\ 0 & 12 \end{bmatrix}$	M1A1		M1 if zeroes appear in the right places
	$= 12\mathbf{I}$	A1F	3	ft provided of right form
(ii)	$q\cos 60° = \frac{1}{2}q = \sqrt{3} \Rightarrow q = 2\sqrt{3}$	M1A1		OE **SC** $q = 2\sqrt{3}$ NMS 1/3
	Other entries verified	E1	3	surd for sin 60º needed
(b)(i)	SF = $q = 2\sqrt{3}$	B1F	1	ft wrong value for q
(ii)	Equation is $y = x \tan 30°$	B1	1	
(c)	$\mathbf{M}^4 = 144\mathbf{I}$	B1F		PI; ft wrong value in (a)(i)
	$\mathbf{M}^4$ gives enlargement SF 144	B1F	2	ft if c's $\mathbf{M}^4 = k\mathbf{I}$
	Total		**10**	

MFP1 (cont)

Q	Solution	Marks	Totals	Comments
7(a)(i)	$(-1+h)^3 = -1+3h-3h^2+h^3$	B1		PI
	$y_B = (-1+3h-3h^2+h^3)+1-h+1$	B1F		ft numerical error
	$\ldots = 1+2h-3h^2+h^3$	B1	3	convincingly shown (AG)
(ii)	Subtraction of 1 and division by h	M1M1		
	Gradient of chord = $2-3h+h^2$	A1	3	
(iii)	As $h \to 0$, gr(chord) $\to$ gr(tgt) = 2	E1B1F	2	E0 if '$h = 0$' used; ft wrong value of p
(b)(i)	$x_2 = -1 - \frac{1}{2} = -1.5$	M1		
		A1F	2	ft wrong gradient
(ii)	Tangent at A drawn	M1		
	α and x_2 shown correctly	A1	2	dep't only on the last M1
	Total		**12**	
8(a)(i)	$\alpha+\beta = 2$, $\alpha\beta = 4$	B1B1		
	$\alpha^3+\beta^3 = (2)^3 - 3(4)(2) = -16$	M1A1		
	$\alpha^3\beta^3 = (4)^3 = 64$, hence result	M1A1	6	convincingly shown (AG)
(ii)	Discriminant 0, so roots equal	B1E1	2	or by factorisation
(b)	$x = \dfrac{2 \pm \sqrt{4-16}}{2}$	M1		or by completing square
	$\ldots = 1 \pm \frac{1}{2}\mathrm{i}\sqrt{12}$	A1	2	
(c)	$\alpha, \beta = 1 \pm \mathrm{i}\sqrt{3}$			
	and $\alpha^3 = \beta^3$, hence result	E2	2	
	Total		**12**	
9(a)	Asymptotes $x = 0$, $x = 4$, $y = 0$	B1 × 3	3	
(b)	$y = k \Rightarrow 2 = kx(x-4)$	M1		
	$\ldots \Rightarrow 0 = kx^2 - 4kx - 2$	A1		
	Discriminant = $(4k)^2 + 8k$	m1		
	At SP $y = -\frac{1}{2}$	A1		not just $k = -\frac{1}{2}$
	$\ldots \Rightarrow 0 = -\frac{1}{2}x^2 + 2x - 2$	m1		
	So $x = 2$	A1	6	
(c)		B1		Curve with three branches approaching vertical asymptotes correctly
		B1		Outer branches correct
		B1	3	Middle branch correct
	Total		**12**	
	TOTAL		**75**	

Further pure 1 - AQA - January 2008

Question 1:

$$z_1 = 2+i \qquad z_1^* = 2-i$$

$$x+3iy = z_1 + 4iz_1^*$$

$$x+3iy = 2+i+4i(2-i)$$

$$x+3iy = 6+9i \qquad so\ x=6$$

$$y=3$$

The great majority of candidates found this question a good starter and obtained full marks without too much trouble. Careless errors sometimes caused candidates to miss out on one of the method marks for the question.

Question 2:

$$Euler\ \ formula: y_{n+1} = y_n + hf(x_n) \qquad with\ \ f(x) = 2^x \ \ and\ \ h = 0.01$$

$$x_1 = 1,\ \ y_1 = 4 \quad so$$

$$for\ \ x_2 = 1+0.01 = 1.01, \qquad y_2 = 4+0.01\times 2^1 = 4.02$$

$$for\ \ x_3 = 1.002, \qquad y_3 = 4.02 + 0.01\times 2^{1.01} = 4.04014 \ \ correct\,to\,5\,sig.\,fig.$$

This question provided most candidates with a further five marks. The most common error was to carry out three iterations instead of only two, which would usually cause the loss of only one mark as long as the working was fully shown; though of course the candidate may well have lost valuable time carrying out the unwanted calculations.

Question 3:

$$Tan\,4(x-\frac{\pi}{8}) = 1 = Tan\frac{\pi}{4}$$

$$so \qquad 4x - \frac{\pi}{2} = \frac{\pi}{4} + k\pi$$

$$4x = \frac{3\pi}{4} + k\pi$$

$$x = \frac{3\pi}{16} + k\frac{\pi}{4} \qquad k\in\mathbb{Z}$$

As usual in MFP1, many candidates were not thoroughly prepared for the task of finding the general solution of a trigonometric equation. The most common approach was to find (usually correctly) one value of *x* and then to add a term *nπ* to this value. Many candidates showed only a slight degree of familiarity with radians, and there were some cases of serious misunderstanding of the implied order of operations in the expression $\tan 4\left(x-\frac{\pi}{8}\right)$

Question 4:

$$a)\sum_{r=1}^{n} r^3 - 6r = \sum_{r=1}^{n} r^3 - 6\sum_{r=1}^{n} r = \frac{1}{4}n^2(n+1)^2 - 6\times\frac{1}{2}n(n+1)$$

$$= \frac{1}{4}n(n+1)\left[n(n+1)-12\right]$$

$$= \frac{1}{4}n(n+1)\left(n^2+n-12\right)$$

$$\sum_{r=1}^{n} r^3 - 6r = \frac{1}{4}n(n+1)(n+4)(n-3)$$

$$b)\, S = \sum_{r=1}^{1000} r^3 - 6r = \frac{1}{4}\times 1000\times 1001\times 1004\times 997 =$$

$$= \frac{1}{4}\times 500\times 2\times 1001\times 1004\times 997 = 2008\times 125\times 1001\times 997$$

$$which\ \ is\ \ a\ \ multiple\ \ of\ \ 2008.$$

Part (a) As in the previous question, many candidates were not sufficiently familiar with the techniques needed to carry out the necessary manipulation efficiently. It was noticeable that many candidates still obtained full marks despite their failure to spot the quick method of taking out common factors at the earliest opportunity. A common mistake was to omit the numerical factor $\frac{1}{4}$, or to replace it with some other number such as 4.
Part (b) Good attempts at this part of the question were few and far between. Many candidates made no attempt at all. Some came to a halt after replacing *n* by 1000 in their answer to part (a). Those who found a factor 1004 usually went on to try to explain how this would lead to a multiple of 2008, but more often than not their arguments lacked cogency.

Question 5:

$a)\ Asymptotes:\ y=\frac{1}{2}x\ \ and\ \ y=-\frac{1}{2}x$

$b)$ when $x=4,\ 4-y^2=1 \qquad y^2=3$

$so\ \ A(4,\sqrt{3})\ and\ \ B(4,-\sqrt{3})$

$c)i)$ Image of A: $(4, 2+\sqrt{3})$

Image of B: $(4, 2-\sqrt{3})$

$ii)\ Equation\ of\ the\ hyperbola\ after\ translation: \frac{x^2}{4}-(y-2)^2=1$

$Equations\ of\ the\ asymptotes:\ y=\pm\frac{1}{2}x+2$

The oblique asymptotes of a hyperbola were not always known by the candidates, though many found the necessary general equations in the formula booklet and correctly applied them to this particular case. Part (b) was very well answered by almost all candidates, while part (c) usually provided some further marks, the most common error being to use $y + 2$ instead of $y - 2$ in the equations of the translated hyperbola and asymptotes.

Question 6:

$a)i) M=\begin{bmatrix}\sqrt{3} & 3\\ 3 & -\sqrt{3}\end{bmatrix}\ and\ \ M^2=\begin{bmatrix}12 & 0\\ 0 & 12\end{bmatrix}=12I \qquad p=12$

$ii) Cos60^o=\frac{1}{2}\quad so\quad M=2\sqrt{3}\begin{bmatrix}\frac{1}{2} & \frac{\sqrt{3}}{2}\\ \frac{\sqrt{3}}{2} & -\frac{1}{2}\end{bmatrix}=2\sqrt{3}\begin{bmatrix}Cos60^o & Sin60^o\\ Sin60^o & -Cos60^o\end{bmatrix}$

$q=2\sqrt{3}$

$b)i)\ Scale\ \ factor\ is\ q=2\sqrt{3}$

$ii)$ Reflection in the line $\ y=(Tan30^o)x$

$y=\frac{1}{\sqrt{3}}x$

$c) M^2=12I\ \ so\ M^4=144I$

This represent an enlargement scale factor 144.

Part (a)(i) of this question was almost universally well answered, but part (a)(ii) led to some very slipshod and unclear reasoning. In part (b)(i) many candidates did not appreciate that the scale factor of the enlargement must be the same as the value of q obtained in the previous part. In part (b)(ii) a common mistake was to use tan 60° in finding the gradient of the mirror line instead of halving the angle to obtain tan 30°. Answers to part (c) were mostly very good, even from candidates who had been struggling with the earlier parts of the question.

Question 7:

$a)\,i)\,y = x^3 - x + 1 \qquad with\ \ x_B = -1 + h$

$gives \quad y_B = (-1+h)^3 - (-1+h) + 1$

$= -1 + 3h - 3h^2 + h^3 + 1 - h + 1 = 1 + 2h - 3h^2 + h^3$

$A(-1,1)\ and\ B(-1+h, 1+2h-3h^2+h^3)$

$ii)\,Gradient\ of\ \ AB = \dfrac{(1+2h-3h^2+h^3)-1}{(-1+h)-(-1)}$

$= \dfrac{2h-3h^2+h^3}{h} = 2 - 3h + h^2$

$iii)$ The gradient of the tangent is

the limit of the gradient of the chord when h tends to 0

$2 - 3h + h^2 \xrightarrow[h \to 0]{} 2$

The gradient of the tangent is 2

b) $i)$ If x_1 is an approximation of the root α,

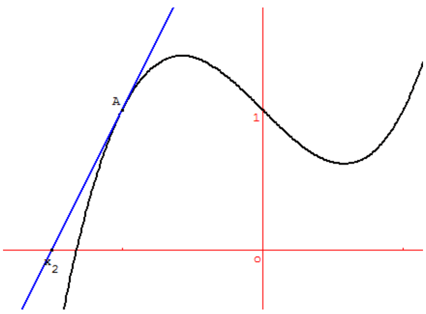
then $x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$ is a better aproximation

$x_1 = -1, \quad f(x_1) = f(-1) = 1$

$f'(x) = 3x^2 - 1\ \ and\ \ f'(-1) = 3 - 1 = 2$

$so\ \ x_2 = -1 - \dfrac{1}{2} = -1.5$

$ii)$



Most candidates performed well in this question.

Part (a)(i) seemed to present a stiff challenge to their algebraic skill, though they often came through the challenge successfully after several lines of working. Part (a)(ii) again proved harder than the examiners had intended, some candidates having to struggle to evaluate the *y*-coordinate of the point *A*. Again the outcome was usually successful, though a substantial minority of candidates used differentiation of the answer to the previous part. In part (a)(iii) it was pleasing to see that a very good proportion of the candidates correctly mentioned *h* 'tending to' zero rather than 'being equal to' zero, which was not allowed, though the correct value of the gradient could be obtained by this method and one mark was awarded for this.

The responses to part (b) suggested that most candidates were familiar with the Newton-Raphson method and were able to apply it correctly, but that they lacked an understanding of the geometry underlying the method, so that they failed to draw a tangent at the point *A* on the insert as required, or failed to indicate correctly the relationship between this tangent and the *x*-axis.

Question 8:

$a)\,i)\,x^2 - 2x + 4 = 0\ has\ roots\ \alpha\ and\ \beta$

$\alpha + \beta = 2 \qquad \alpha\beta = 4$

$\alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta) \qquad \alpha^3\beta^3 = (\alpha\beta)^3 = 4^3 = 64$

$= 2^3 - 3 \times 4 \times 2 = -16$

Therefore an equation with roots $\alpha^3\ and\ \beta^3\ is\ \ x^2 + 16x + 64 = 0$

$ii)$ Discriminant $= 16^2 - 4 \times 1 \times 64 = 256 - 256 = 0$

The roots are real and equal

$b)\ x^2 - 2x + 4 = 0$

discriminant $= (-2)^2 - 4 \times 1 \times 4 = 4 - 16 = -12 = (2i\sqrt{3})^2$

$x = \dfrac{2 \pm 2i\sqrt{3}}{2} \qquad x = 1 \pm i\sqrt{3}$

$c)\,We\ can\ call\ \alpha = 1 + i\sqrt{3}\ and\ \beta = 1 - i\sqrt{3}$

$from\ the\ question\ a)ii),\ we\ know\ that\ \alpha^3 = \beta^3$

$meaning\ \ (1+i\sqrt{3})^3 = (1-i\sqrt{3})^3$

This question proved to be an excellent source of marks for the majority of candidates, apart from the discriminating test provided by part (c). Many candidates seemed to be familiar with the techniques relating to the cubes of the roots of a quadratic equation, though some struggled to find the correct expression for the sum of the cubes of the roots, while others lost marks by not showing enough evidence in view of the fact that the required equation was printed on the question paper.

Parts (a)(ii) and (b) proved straightforward for most candidates, but only a few were able to give a clear explanation in part (c), where many candidates claimed that the original roots *α* and *β* must be equal.

Question 9:

$$y=\frac{2}{x(x-4)}$$

$a)$"*Vertical asymptotes*" $x=0$ *and* $x=4$ (*roots of the* denominator)

$$y=\frac{2}{x^2-4x}=\frac{\frac{2}{x^2}}{1-\frac{4}{x}}\xrightarrow[x\to\infty]{}0 \qquad y=0 \text{ is asymptote to the curve}$$

$b)$ $y=\dfrac{2}{x^2-4x}=k$ *can be re-arranged to* $kx^2-4kx-2=0$

Discriminant: $(-4k)^2-4\times k\times(-2)=16k^2+8k=8k(2k+1)$

the line $y=k$ is tangent to the curve when the discriminant is 0

this gives $k=0$ (*impossible, for all* $x, \dfrac{2}{x^2-4x}\neq 0$) or $k=-\dfrac{1}{2}$

And for $k=-\dfrac{1}{2}$, the equation becomes $-\dfrac{1}{2}x^2+2x-2=0$

$$x^2-4x+4=0$$

$$(x-2)^2=0$$

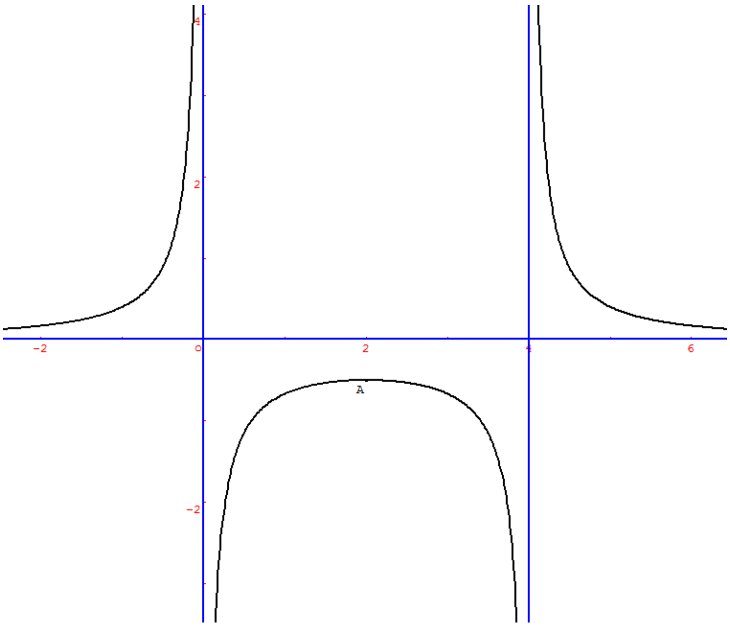
$$x=2$$

The stationary point has coordinates $(2,-\frac{1}{2})$

Most candidates scored well in parts (a) and (c) of this question. Many wrote down the equations of the two vertical asymptotes without any apparent difficulty, but struggled to find the horizontal asymptote, although in most cases they were successful. The sketch-graph in part (c) was usually drawn correctly.

Part (b) was a standard exercise for the more able candidates. No credit was given for asserting that because $x = 0$ and $x = 4$ were asymptotes, it followed that $x = 2$ must provide a stationary point. A carefully reasoned argument based on the symmetry of the function in the denominator would have earned credit, but most candidates quite reasonably preferred to adopt the standard approach.

$c)$



Grade boundaries

GCE

Component Code	Component Title	Maximum Scaled Mark	A	B	C	D	E
MFP1	GCE MATHEMATICS UNIT FP1	75	61	53	45	37	30

General Certificate of Education
June 2008
Advanced Subsidiary Examination

MATHEMATICS **MFP1**
Unit Further Pure 1

Monday 16 June 2008 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables
- an insert for use in Questions 4 and 8 (enclosed).

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- Fill in the boxes at the top of the insert.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The equation

$$x^2 + x + 5 = 0$$

has roots α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) Find the value of $\alpha^2 + \beta^2$. *(2 marks)*

(c) Show that $\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha} = -\dfrac{9}{5}$. *(2 marks)*

(d) Find a quadratic equation, with integer coefficients, which has roots $\dfrac{\alpha}{\beta}$ and $\dfrac{\beta}{\alpha}$. *(2 marks)*

2 It is given that $z = x + \mathrm{i}y$, where x and y are real numbers.

(a) Find, in terms of x and y, the real and imaginary parts of

$$3\mathrm{i}z + 2z^*$$

where z^* is the complex conjugate of z. *(3 marks)*

(b) Find the complex number z such that

$$3\mathrm{i}z + 2z^* = 7 + 8\mathrm{i}$$ *(3 marks)*

3 For each of the following improper integrals, find the value of the integral **or** explain briefly why it does not have a value:

(a) $\displaystyle\int_9^\infty \frac{1}{\sqrt{x}}\,\mathrm{d}x$; *(3 marks)*

(b) $\displaystyle\int_9^\infty \frac{1}{x\sqrt{x}}\,\mathrm{d}x$. *(4 marks)*

4 [**Figure 1** and **Figure 2**, printed on the insert, are provided for use in this question.]

The variables x and y are related by an equation of the form

$$y = ax + \frac{b}{x+2}$$

where a and b are constants.

(a) The variables X and Y are defined by $X = x(x+2)$, $Y = y(x+2)$.

Show that $Y = aX + b$. *(2 marks)*

(b) The following approximate values of x and y have been found:

x	1	2	3	4
y	0.40	1.43	2.40	3.35

(i) Complete the table in **Figure 1**, showing values of X and Y. *(2 marks)*

(ii) Draw on **Figure 2** a linear graph relating X and Y. *(2 marks)*

(iii) Estimate the values of a and b. *(3 marks)*

5 (a) Find, in **radians**, the general solution of the equation

$$\cos\left(\frac{x}{2} + \frac{\pi}{3}\right) = \frac{1}{\sqrt{2}}$$

giving your answer in terms of π. *(5 marks)*

(b) Hence find the smallest **positive** value of x which satisfies this equation. *(2 marks)*

6 The matrices **A** and **B** are given by

$$\mathbf{A} = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$$

(a) Calculate the matrix **AB**. *(2 marks)*

(b) Show that $\mathbf{A}^2$ is of the form $k\mathbf{I}$, where k is an integer and $\mathbf{I}$ is the 2×2 identity matrix. *(2 marks)*

(c) Show that $(\mathbf{AB})^2 \neq \mathbf{A}^2\mathbf{B}^2$. *(3 marks)*

7 A curve C has equation

$$y = 7 + \frac{1}{x+1}$$

(a) Define the translation which transforms the curve with equation $y = \frac{1}{x}$ onto the curve C. *(2 marks)*

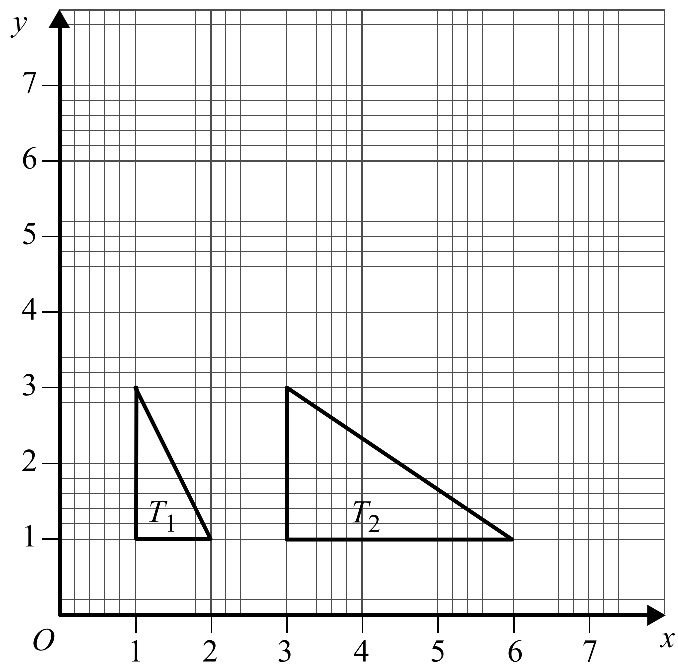
(b) (i) Write down the equations of the two asymptotes of C. *(2 marks)*

(ii) Find the coordinates of the points where the curve C intersects the coordinate axes. *(3 marks)*

(c) Sketch the curve C and its two asymptotes. *(3 marks)*

8 [**Figure 3**, printed on the insert, is provided for use in this question.]

The diagram shows two triangles, T_1 and T_2.



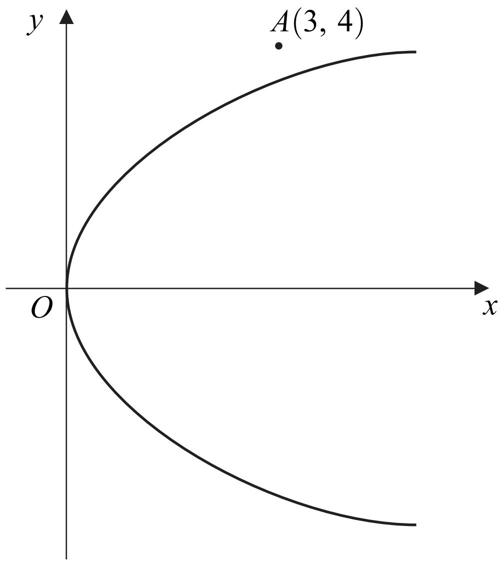
(a) Find the matrix of the stretch which maps T_1 to T_2. *(2 marks)*

(b) The triangle T_2 is reflected in the line $y = x$ to give a third triangle, T_3.

On **Figure 3**, draw the triangle T_3. *(2 marks)*

(c) Find the matrix of the transformation which maps T_1 to T_3. *(3 marks)*

9 The diagram shows the parabola $y^2 = 4x$ and the point A with coordinates $(3, 4)$.



(a) Find an equation of the straight line having gradient m and passing through the point $A(3, 4)$. *(2 marks)*

(b) Show that, if this straight line intersects the parabola, then the y-coordinates of the points of intersection satisfy the equation

$$my^2 - 4y + (16 - 12m) = 0$$ *(3 marks)*

(c) By considering the discriminant of the equation in part (b), find the equations of the two tangents to the parabola which pass through A.

(No credit will be given for solutions based on differentiation.) *(5 marks)*

(d) Find the coordinates of the points at which these tangents touch the parabola. *(4 marks)*

END OF QUESTIONS

Surname		Other Names	
Centre Number		Candidate Number	
Candidate Signature			

General Certificate of Education
June 2008
Advanced Subsidiary Examination

MATHEMATICS **MFP1**
Unit Further Pure 1

Insert

Insert for use in **Questions 4 and 8**.

Fill in the boxes at the top of this page.

Fasten this insert securely to your answer book.

Figure 1 (for use in Question 4)

x	1	2	3	4
y	0.40	1.43	2.40	3.35
X	3			
Y	1.20			

Figure 2 (for use in Question 4)

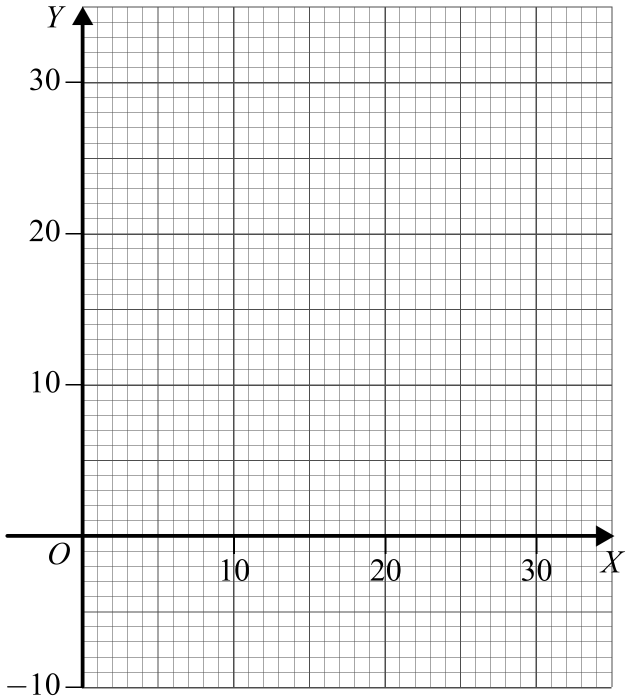
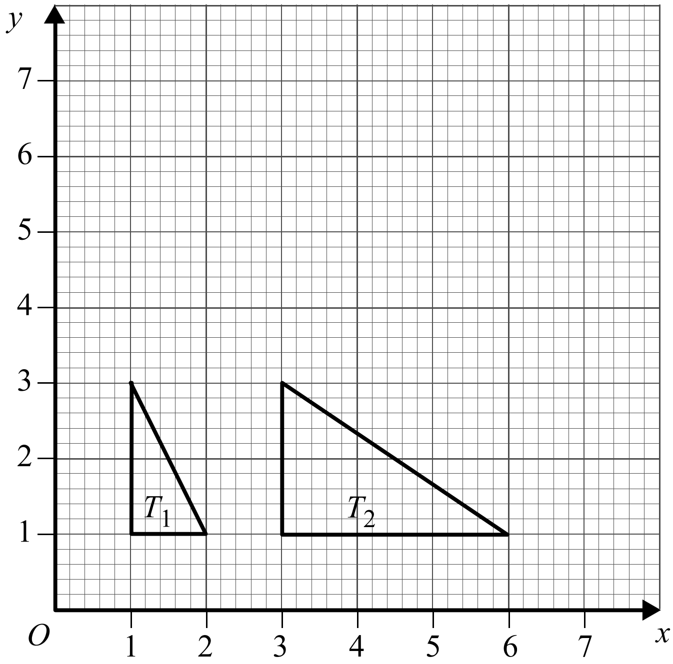


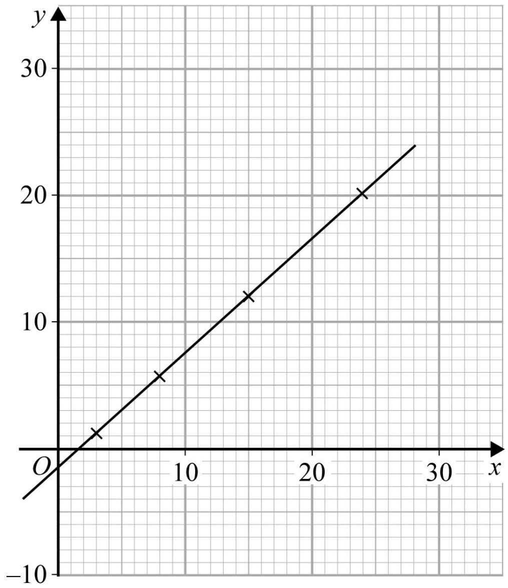
Figure 3 (for use in Question 8)



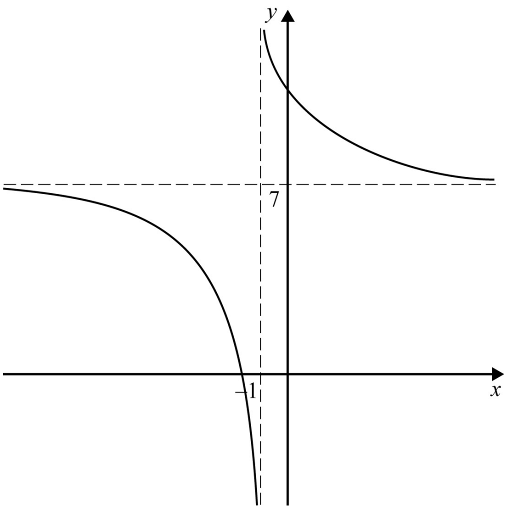
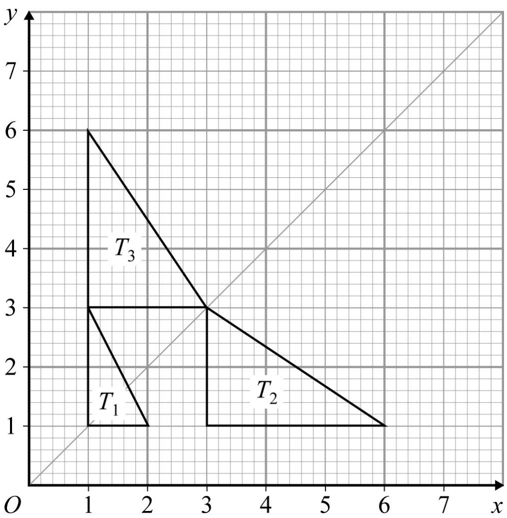
MFP1

Q	Solution	Marks	Total	Comments
1(a)	$\alpha + \beta = -1$, $\alpha\beta = 5$	B1B1	2	
(b)	$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$	M1		with numbers substituted
	$\ldots = 1 - 10 = -9$	A1F	2	ft sign error(s) in (a)
(c)	$\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha} = \dfrac{\alpha^2 + \beta^2}{\alpha\beta}$	M1		
	$\ldots = -\dfrac{9}{5}$	A1	2	AG: A0 if $\alpha + \beta = 1$ used
(d)	Product of new roots is 1	B1		PI by constant term 1 or 5
	Eqn is $5x^2 + 9x + 5 = 0$	B1F	2	ft wrong value for product
	Total		**8**	
2(a)	Use of $z^* = x - \mathrm{i}y$	M1		
	Use of $\mathrm{i}^2 = -1$	M1		
	$3\mathrm{i}z + 2z^* = (2x - 3y) + \mathrm{i}\,(3x - 2y)$	A1	3	Condone inclusion of i in I part
(b)	Equating R and I parts	M1		
	$2x - 3y = 7,\ \ 3x - 2y = 8$	m1		with attempt to solve
	$z = 2 - \mathrm{i}$	A1	3	Allow $x = 2$, $y = -1$
	Total		**6**	
3(a)	$\int x^{-\frac{1}{2}}\ \mathrm{d}x = 2x^{\frac{1}{2}}(+c)$	M1A1		M1 for correct power in integral
	$x^{\frac{1}{2}} \to \infty$ as $x \to \infty$, so no value	E1	3	
(b)	$\int x^{-\frac{3}{2}}\ \mathrm{d}x = -2x^{-\frac{1}{2}}(+c)$	M1A1		M1 for correct power in integral
	$x^{-\frac{1}{2}} \to 0$ as $x \to \infty$	E1		PI
	$\int_9^\infty x^{-\frac{3}{2}}\ \mathrm{d}x = -2(0 - \frac{1}{3}) = \frac{2}{3}$	A1	4	Allow A1 for correct answer even if not fully explained
	Total		**7**	
4(a)	Multiplication by $x + 2$	M1		applied to all 3 terms
	$Y = aX + b$ convincingly shown	A1	2	AG
(b)(i)	$X = 8, 15, 24$ in table	B1		
	$Y = 5.72, 12, 20.1$ in table	B1	2	Allow correct to 2SF

MFP1 (cont)

Q	Solution	Marks	Total	Comments
4(b)(ii)				
	Four points plotted	B1F		ft incorrect values in table
	Reasonable line drawn	B1F	2	ft incorrect points
(iii)	Method for gradient	M1		or algebraic method for a or b
	a = gradient ≈ 0.9	A1		Allow from 0.88 to 0.93 incl
	b = Y-intercept ≈ -1.5	B1F	3	Allow from -2 to -1 inclusive; ft incorrect points/line **NMS** B1 for a, B1 for b
	Total		**9**	
5(a)	$\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ stated or used	B1		Degrees or decimals penalised in 5th mark only
	Appropriate use of $\pm$	B1		OE
	Introduction of $2n\pi$	M1		OE
	Subtraction of $\frac{\pi}{3}$ and multiplication by 2	m1		All terms multiplied by 2
	$x = -\frac{2\pi}{3} \pm \frac{\pi}{2} + 4n\pi$	A1	5	OE
5(b)	$n = 1$ gives min pos $x = \frac{17\pi}{6}$	M1A1	2	**NMS** 1/2 provided (a) correct
	Total		**7**	
6(a)	$\mathbf{AB} = \begin{bmatrix} 0 & -4 \\ 4 & 0 \end{bmatrix}$	M1A1	2	M1A0 if 3 entries correct
(b)	$\mathbf{A}^2 = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$	B1		
	$\ldots = 4\mathbf{I}$	B1	2	
(c)	$(\mathbf{AB})^2 = -16\mathbf{I}$	B1		
	$\mathbf{B}^2 = 4\mathbf{I}$	B1		PI
	so $\mathbf{A}^2\,\mathbf{B}^2 = 16\mathbf{I}$ (hence result)	B1	3	Condone absence of conclusion
	Total		**7**	

MFP1 (cont)

Q	Solution	Marks	Total	Comments
7(a)	Curve translated 7 in y direction ... and 1 in negative x direction	B1 B1	2	or answer in vector form
(b)(i)	Asymptotes $x = -1$ and $y = 7$	B1B1	2	
(ii)	Intersections at (0, 8) ... ... and $(-\frac{8}{7}, 0)$	B1 M1A1	3	Allow AWRT –1.14; NMS 1/2
(c)	 At least one branch Complete graph All correct including asymptotes	B1 B1 B1	3	of correct shape translation of $y = 1/x$ in roughly correct positions
	Total		**10**	
8(a)	Matrix is $\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$	M1A1	2	M1 if zeros in correct positions; allow NMS
(b)	 Third triangle shown correctly	M1A1	2	M1A0 if one point wrong

MFP1 (cont)

Q	Solution	Marks	Total	Comments
8(c)	Matrix of reflection is $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	B1		**Alt**: calculating matrix from the coordinates: M1 A2,1
	Multiplication of above matrices	M1		in correct order
	Answer is $\begin{bmatrix} 0 & 1 \\ 3 & 0 \end{bmatrix}$	A1F	3	ft wrong answer to (a); **NMS** 1/3
	Total		**7**	
9(a)	Equation is $y - 4 = m(x - 3)$	M1A1	2	OE; M1A0 if one small error
(b)	Elimination of x	M1		
	$4y - 16 = m(y^2 - 12)$	A1		OE (no fractions)
	Hence result	A1	3	convincingly shown (AG)
(c)	Discriminant equated to zero	M1		
	$(3m - 1)(m - 1) = 0$	m1A1		OE; m1 for attempt at solving
	Tangents $y = x + 1$, $y = \frac{1}{3}x + 3$	A1A1	5	OE
(d)	$m = 1 \Rightarrow y^2 - 4y + 4 = 0$	M1		OE; $m = 1$ needed for this
	so point of contact is (1, 2)	A1		
	$m = \frac{1}{3} \Rightarrow \frac{1}{3}y^2 - 4y + 12 = 0$	M1		OE; $m = \frac{1}{3}$ needed for this
	so point of contact is (9, 6)	A1	4	
	Total		**14**	
	TOTAL		**75**	

Further pure 1 - AQA - June 2008

Question 1:

$x^2+x+5=0$ *has roots* α *and* β

$a)\ \alpha+\beta=-1 \quad and \quad \alpha\beta=5$

$b)\ \alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=(-1)^2-2\times5=1-10=-9$

$$c)\ \frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^2+\beta^2}{\alpha\beta}=\frac{-9}{5}$$

$$d)\ \frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{-9}{5} \quad and \quad \frac{\alpha}{\beta}\times\frac{\beta}{\alpha}=\frac{\alpha\beta}{\alpha\beta}=1$$

so an equation with roots $\frac{\alpha}{\beta}$ *and* $\frac{\beta}{\alpha}$ *is* $x^2+\frac{9}{5}x+1=0$

$$5x^2+9x+5=0$$

The great majority of candidates showed a confident grasp of the algebra needed to deal with the sum of the squares of the roots of a quadratic equation, and answered all parts of this question efficiently. The only widespread loss of credit came in the very last part, where many candidates failed to give integer coefficients or else found a quadratic expression but without the necessary '= 0' to make it an equation.

Question 2:

$z=x+iy$

$a)\ 3iz+2z^*=3i(x+iy)+2(x-iy)=3ix-3y+2x-2iy$

$\qquad =(2x-3y)+i(3x-2y)$

$\mathrm{Re}(3iz+2z^*)=2x-3y$

$\mathrm{Im}(3iz+2z^*)=3x-2y$

$$b)\ 3iz+2z^*=7+8i \quad means \quad \begin{cases}2x-3y=7\\3x-2y=8\end{cases} \quad \begin{cases}6x-9y=21\\6x-4y=16\end{cases} \quad \begin{cases}5y=-5\\2x-3y=7\end{cases}$$

$y=-1 \quad and \quad x=2$

The solution is $2-i$

Here again most candidates answered confidently and accurately. In part (a) many failed to state clearly which was the real part and which the imaginary part, though they recovered ground by using the correct expressions in part (b). As already stated, the solution of the simultaneous equations was often attempted by a substitution method. Whichever method was used, numerical and sign errors were fairly common, but most candidates obtained the correct values for *x* and *y*. Two faults which were condoned this time were, in part (a), the retention of the factor ' i ' in the imaginary part, and, in part (b) following correct values of *x* and *y*, a failure to give the final value of *z* correctly

Question 3:

$$a)\int \frac{1}{\sqrt{x}}dx=\int x^{-\frac{1}{2}}dx=2\sqrt{x}+c$$

$$2\sqrt{x}\xrightarrow[x\to\infty]{}\infty$$

This integral has no value.

$$b)\int \frac{1}{x\sqrt{x}}dx=\int x^{-\frac{3}{2}}dx=-2x^{-\frac{1}{2}}+c=\frac{-2}{\sqrt{x}}+c$$

$$\frac{-2}{\sqrt{x}}\xrightarrow[x\to\infty]{}0$$

$$so\ \int_9^\infty \frac{1}{x\sqrt{x}}dx=0-\frac{-2}{\sqrt{9}}=\frac{2}{3}$$

There were many all-correct solutions to this question from the stronger candidates. Others were unsure of themselves when dealing with the behaviour of powers of *x* as *x* tended to infinity. Many others did not reach the stage of making that decision: either they failed to convert the integrands correctly into powers of *x*, or they integrated their powers of *x* incorrectly. A reasonable grasp of AS Pure Core Mathematics is essential for candidates taking this paper.

Question 4:

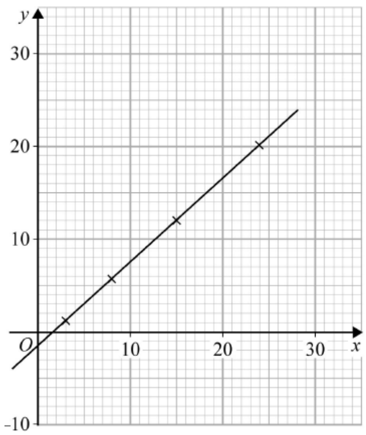
$$a)\ y = ax + \frac{b}{x+2} \qquad (\times(x+2))$$

$$y(x+2) = ax(x+2) + b$$

$$Y = aX + b$$

b)

x	1	2	3	4
y	0.40	1.43	2.40	3.35
X	3	8	15	24
Y	1.20	5.72	12	20.1



$$iii)\ The\ gradient\ is: a \approx \frac{20.1-1.20}{24-3} \qquad a \approx 0.9$$

$$b = Y - aX = 1.20 - 0.9\times 3 = -1.5 \qquad b \approx -1.5$$

In part (a) most candidates simply wrote down the given equation, multiplied through by ($x+2$), and converted the result into X and Y notation. This was all that was required for the award of the two marks, but some candidates thought that more was needed and presented some rather heavy algebra. Some candidates lost credit because of a confusion between the upper- and lower-case letters.

Parts (b)(i) and (b)(ii) were usually answered correctly on the insert, after which most candidates knew how to find estimates for a and b, occasionally losing a mark through a loss of accuracy after a poor choice of coordinates to use in the calculation of the gradient. Another way of losing a mark was to write down the estimate for a without showing any working.

Question 5:

$$Cos\left(\frac{x}{2}+\frac{\pi}{3}\right) = \frac{1}{\sqrt{2}} = Cos\left(\frac{\pi}{4}\right)$$

$$so\quad \frac{x}{2}+\frac{\pi}{3} = \frac{\pi}{4}+k2\pi \quad or \quad \frac{x}{2}+\frac{\pi}{3} = -\frac{\pi}{4}+k2\pi$$

$$\frac{x}{2} = -\frac{\pi}{12}+k2\pi \quad or \quad \frac{x}{2} = -\frac{7\pi}{12}+k2\pi$$

$$x = -\frac{\pi}{6}+k4\pi \quad or \quad x = -\frac{7\pi}{6}+k4\pi$$

$$b) -\frac{\pi}{6}+4\pi = \frac{23\pi}{6} \quad and \quad -\frac{7\pi}{6}+4\pi = \frac{17\pi}{6}$$

The smallest value of x is $\frac{17\pi}{6}$

As usual in MFP1, many candidates made a poor effort at finding the general solution of a trigonometric equation. In this case the solution $x=\frac{\pi}{4}$ was almost always found in part (a), but the second solution $x=-\frac{\pi}{4}$ (or alternative) was relatively rarely seen. Many candidates were aware that when dealing with a cosine they needed to put a plus or minus symbol somewhere but were not sure where exactly it should go. The general term 2nπ usually appeared, but often in the wrong place. In part (b) only the strongest candidates, and not even all of these, were able to obtain any credit here.

Question 6:

$$A = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} \ and\ B \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$$

$$a)\ AB = \begin{bmatrix} 0 & -4 \\ 4 & 0 \end{bmatrix}$$

$$b)\ A^2 = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} = 4I$$

$$c)\ (AB)^2 = \begin{bmatrix} 0 & -4 \\ 4 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & -4 \\ 4 & 0 \end{bmatrix} = \begin{bmatrix} -16 & 0 \\ 0 & -16 \end{bmatrix} = -16I$$

$$A^2B^2 = 4I \times \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} = 16I \qquad so\ \ (AB)^2 \neq A^2B^2$$

This question was very well answered by the majority of candidates, who showed confidence and accuracy in manipulating these simple matrices.

Question 7:

$$y = 7 + \frac{1}{x+1}$$

$a)\, Translation\ vector \begin{pmatrix} -1 \\ 7 \end{pmatrix}$

$b)\, i)\, "Vertical\ asymptote"\ \ x = -1$

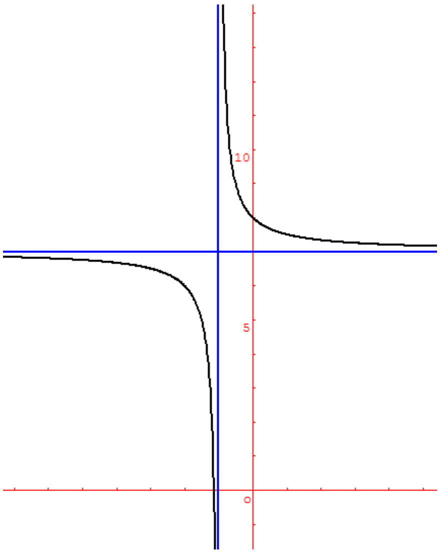
$$y = 7 + \frac{1}{x+1} \xrightarrow[x\to\infty]{} 7 \qquad y = 7 \text{ is asymptote to the curve}$$

$ii)\, When\ x = 0,\ y = 8$

$$y = 0 = 7 + \frac{1}{x+1} \qquad \frac{1}{x+1} = -7 \qquad x+1 = \frac{-1}{7} \qquad x = -\frac{8}{7}$$

The curve crosses the axes at $(0,8)$ *and* $(-\frac{8}{7},0)$

$c)$



Many candidates scored well on this question but, for some, marks were lost in a variety of places.

In part (a) some candidates were careless in giving the two parts of the translation.

In part (b) (i) the horizontal asymptote was often found to be the *x*-axis, which usually caused difficulty in part (c). In part (b)(ii), as in Question 4 (b)(iii), a mark was sometimes lost by a failure to show some necessary working, in this case for the intersection of the curve with the *x*-axis.

In part (c) the sketches were sometimes wildly wrong, despite the information provided in part (a) that the curve must be a translated version of the well-known hyperbola $y = \frac{1}{x}$. In many cases the curve shown was basically correct but did not appear to approach the asymptotes in a satisfactory way. Despite a generous interpretation of this on the part of the examiners, some candidates lost credit because of seriously faulty drawing.

Question 8:

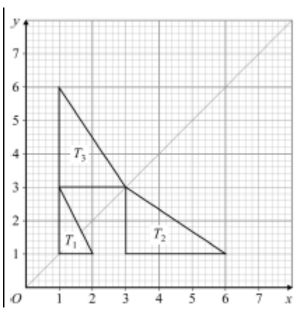
$a)$ This is a stretch in the x-direction by a factor 3

It is represented by the matrix $\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$

$b)$

$c)$ The matrix of the reflection is $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

so The matrix which maps T_1 to T_3 is $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 3 & 0 \end{bmatrix}$



This question was not generally well answered, except for part (b) where most candidates were able to draw the reflected triangle correctly on the insert. In part (a) many candidates seemed not to recognise the transformation as a simple one-way stretch, and even those who did were not always familiar with the corresponding matrix. They could have worked out the matrix by considering the effect of the stretch on the points (1, 0) and (0, 1), but in many cases candidates either gave up, made a wild guess, or embarked on a lengthy algebraic process involving four equations and four unknowns.

In part (c) relatively few candidates saw the benefit of matrix multiplication for the composition of two transformations, and even they often multiplied the matrices the wrong way round. Again it was common to see candidates trying to find the matrix from four equations, but this method was doomed to failure if the candidate, as happened almost every time, paired off the vertices incorrectly.

Question 9:

$y^2 = 4x \qquad\qquad A(3,4)$

$a)\ y-4 = m(x-3) \qquad\qquad y = mx-3m+4$

$b)$ If the line intersects the curve then the coordinates of the point of intersection satisfy both equations simultaneously

$$y^2 = 4x \qquad x = \frac{y^2}{4} \qquad so \quad y = m\frac{y^2}{4} - 3m + 4$$

$$4y = my^2 - 12m + 16$$

$$my^2 - 4y + 16 - 12m = 0$$

$c)$ The line is a tangent when this equation has a repeated root meaning that the discriminant is 0

$$\text{Discriminant} = (-4)^2 - 4 \times m \times (16-12m) = 16 - 64m + 48m^2 = 0$$

$$3m^2 - 4m + 1 = 0$$

$$(3m-1)(m-1) = 0$$

$$m = \frac{1}{3} \quad or \quad m = 1$$

The equation of the two tangents are $y = \frac{1}{3}x + 3 \ \ and \ \ y = x+1$

$d)$ If $m = 1,$ the equation becomes $y^2 - 4y + 4 = 0$

$$(y-2)^2 = 0 \qquad\qquad y = 2 \quad and \quad x = 1$$

If $m = \frac{1}{3},$ the equation becomes $\frac{1}{3}y^2 - 4y + 12 = 0$

$$y^2 - 12y + 36 = 0$$

$$(y-6)^2 = 0 \qquad\qquad y = 6 \quad and \quad x = 9$$

The tangents touch the curve at $(1,2)$ *and* $(9,6)$

Most candidates know that there is likely to be a question involving quadratic theory and are well equipped to answer it. In this case it was possible to answer parts (c) and (d) without having been successful in the earlier parts of the question, and this was often seen. At the same time there were many candidates who did well in parts (a) and (b) but made an error in the discriminant in part (c) leading to a significant loss of marks thereafter.

In part (a) many candidates found a particular value for *m* rather than finding an equation that would be valid for all values of *m*. In part (b), as mentioned above, the elimination was not always carried out by the most efficient method, but many candidates still managed to establish the required equation. In part (c) some candidates lost all credit by failing to indicate that the discriminant must be zero for the line to be a tangent to the parabola; while others found the two gradients correctly but omitted the actual equations asked for in the question.

Those who did find the correct gradients nearly always went on to gain all or most of the marks available in part (d), from a quadratic in *y* (the more direct way) or from a quadratic in *x*

Grade boundaries

GCE

Component Code	Component Title	Maximum Scaled Mark	A	B	C	D	E
			Scaled Mark Grade Boundaries				
MFP1	GCE MATHEMATICS UNIT FP1	75	63	55	48	41	34

General Certificate of Education
January 2009
Advanced Subsidiary Examination

MATHEMATICS **MFP1**
Unit Further Pure 1

Thursday 15 January 2009 9.00 am to 10.30 am

For this paper you must have: • an 8-page answer book • the blue AQA booklet of formulae and statistical tables. You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 A curve passes through the point $(0, 1)$ and satisfies the differential equation

$$\frac{dy}{dx} = \sqrt{1 + x^2}$$

Starting at the point $(0, 1)$, use a step-by-step method with a step length of 0.2 to estimate the value of y at $x = 0.4$. Give your answer to five decimal places. *(5 marks)*

2 The complex number $2 + 3i$ is a root of the quadratic equation

$$x^2 + bx + c = 0$$

where b and c are real numbers.

(a) Write down the other root of this equation. *(1 mark)*

(b) Find the values of b and c. *(4 marks)*

3 Find the general solution of the equation

$$\tan\left(\frac{\pi}{2} - 3x\right) = \sqrt{3}$$ *(5 marks)*

4 It is given that

$$S_n = \sum_{r=1}^{n}(3r^2 - 3r + 1)$$

(a) Use the formulae for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$ to show that $S_n = n^3$. *(5 marks)*

(b) Hence show that $\sum_{r=n+1}^{2n} (3r^2 - 3r + 1) = kn^3$ for some integer k. *(2 marks)*

5 The matrices **A** and **B** are defined by

$$\mathbf{A} = \begin{bmatrix} k & k \\ k & -k \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} -k & k \\ k & k \end{bmatrix}$$

where k is a constant.

(a) Find, in terms of k:

(i) $\mathbf{A} + \mathbf{B}$; *(1 mark)*

(ii) $\mathbf{A}^2$. *(2 marks)*

(b) Show that $(\mathbf{A} + \mathbf{B})^2 = \mathbf{A}^2 + \mathbf{B}^2$. *(4 marks)*

(c) It is now given that $k = 1$.

(i) Describe the geometrical transformation represented by the matrix $\mathbf{A}^2$. *(2 marks)*

(ii) The matrix **A** represents a combination of an enlargement and a reflection. Find the scale factor of the enlargement and the equation of the mirror line of the reflection. *(3 marks)*

6 A curve has equation

$$y = \frac{(x-1)(x-3)}{x(x-2)}$$

(a) (i) Write down the equations of the three asymptotes of this curve. *(3 marks)*

(ii) State the coordinates of the points at which the curve intersects the x-axis. *(1 mark)*

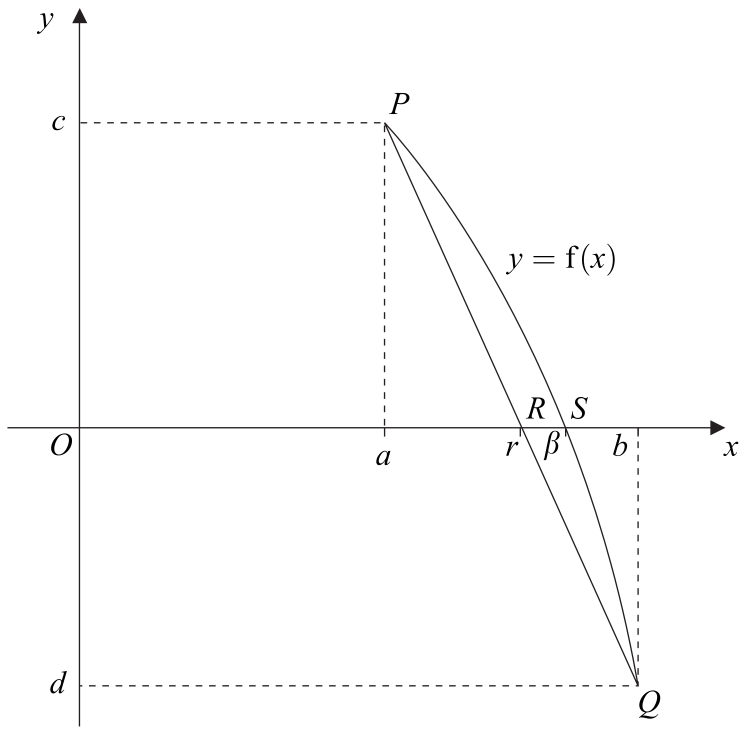
(iii) Sketch the curve.

(You are given that the curve has no stationary points.) *(4 marks)*

(b) Hence, or otherwise, solve the inequality

$$\frac{(x-1)(x-3)}{x(x-2)} < 0$$ *(2 marks)*

7 The points $P(a, c)$ and $Q(b, d)$ lie on the curve with equation $y = \mathrm{f}(x)$. The straight line PQ intersects the x-axis at the point $R(r, 0)$. The curve $y = \mathrm{f}(x)$ intersects the x-axis at the point $S(\beta, 0)$.



(a) Show that

$$r = a + c\left(\frac{b-a}{c-d}\right)$$ *(4 marks)*

(b) Given that

$$a = 2,\ b = 3 \text{ and } \mathrm{f}(x) = 20x - x^4$$

(i) find the value of r; *(3 marks)*

(ii) show that $\beta - r \approx 0.18$. *(3 marks)*

8 For each of the following improper integrals, find the value of the integral **or** explain why it does not have a value:

(a) $\int_1^\infty x^{-\frac{3}{4}}\,dx$; *(3 marks)*

(b) $\int_1^\infty x^{-\frac{5}{4}}\,dx$; *(3 marks)*

(c) $\int_1^\infty \left(x^{-\frac{3}{4}} - x^{-\frac{5}{4}}\right)dx$. *(1 mark)*

9 A hyperbola H has equation

$$x^2 - \frac{y^2}{2} = 1$$

(a) Find the equations of the two asymptotes of H, giving each answer in the form $y = mx$. *(2 marks)*

(b) Draw a sketch of the two asymptotes of H, using roughly equal scales on the two coordinate axes. Using the same axes, sketch the hyperbola H. *(3 marks)*

(c) (i) Show that, if the line $y = x + c$ intersects H, the x-coordinates of the points of intersection must satisfy the equation

$$x^2 - 2cx - (c^2 + 2) = 0$$ *(4 marks)*

(ii) Hence show that the line $y = x + c$ intersects H in two distinct points, whatever the value of c. *(2 marks)*

(iii) Find, in terms of c, the y-coordinates of these two points. *(3 marks)*

END OF QUESTIONS

MFP1

Q	Solution	Marks	Total	Comments
1	First increment is 0.2, so $y \approx 1.2$	B1B1		PI; variations possible here
	Second increment is $0.2\sqrt{1+0.2^2}$	M1		
	... $\approx 0.203\ 961$, so $y \approx 1.403\ 96$	A2,1F	5	A1 if accuracy lost; ft num error
	Total		**5**	
2(a)	Other root is $2 - 3\mathrm{i}$	B1	1	
(b)	Sum of roots = 4	B1F		ft error in (a)
	So $b = -4$	B1F		ft wrong value for sum
	Product is 13	B1		
	So $c = 13$	B1F	4	ft wrong value for product
	Alternative:			
	Substituting $2 + 3\mathrm{i}$ into equation	M1		
	Equating R and I parts	m1		
	$12 + 3b = 0$, so $b = -4$	A1		
	$-5 + 2b + c = 0$, so $c = 13$	A1F	(4)	ft wrong value for b
	Total		**5**	
3	$\tan\frac{\pi}{3} = \sqrt{3}$	B1		Decimals/degrees penalised at 5^{th} mark
	Introduction of $n\pi$	M1		(or $2n\pi$) at any stage
	Going from $\frac{\pi}{2} - 3x$ to x	m1		Including dividing all terms by 3
	$x = \frac{\pi}{18} + \frac{1}{3}n\pi$	A2,1F	5	Allow +, − or ±; A1 with dec/deg; ft wrong first solution
	Total		**5**	
4(a)	$S_n = 3\Sigma r^2 - 3\Sigma r + \Sigma 1$	M1		
	Correct expressions substituted	m1		At least for first two terms
	Correct expansions	A1		
	$\Sigma 1 = n$	B1		
	Answer convincingly obtained	A1	5	AG
(b)	$S_{2n} - S_n$ attempted	M1		Condone $S_{2n} - S_{n+1}$ here
	Answer $7n^3$	A1	2	
	Total		**7**	

MFP1 (cont)

Q	Solution	Marks	Total	Comments
5(a)(i)	$\mathbf{A}+\mathbf{B}=\begin{bmatrix}0 & 2k\\ 2k & 0\end{bmatrix}$	B1	1	
(ii)	$\mathbf{A}^2=\begin{bmatrix}2k^2 & 0\\ 0 & 2k^2\end{bmatrix}$	B2,1	2	B1 if three entries correct
(b)	$(\mathbf{A}+\mathbf{B})^2 = =\begin{bmatrix}4k^2 & 0\\ 0 & 4k^2\end{bmatrix}$	B2,1		B1 if three entries correct
	$\mathbf{B}^2=\mathbf{A}^2$, hence result	B1B1	4	
(c)(i)	$\mathbf{A}^2$ is an enlargement (centre O) with SF 2	M1 A1	 2	 Condone $2k^2$
(ii)	Scale factor is now $\sqrt{2}$ Mirror line is $y=x\tan 22\frac{1}{2}°$	B1 M1A1	 3	Condone $\sqrt{2}k$
	Total		**12**	
6(a)(i)	Asymptotes $x=0$, $x=2$, $y=1$	B1×3	3	
(ii)	Intersections at (1, 0) and (3, 0)	B1	1	
(iii)	At least one branch approaching asymptotes	B1		
	Each branch	B1×3	4	
(b)	$0<x<1$, $2<x<3$	B1,B1	2	Allow B1 if one repeated error occurs, eg ≤ for <
	Alternative: Complete correct algebraic method	 M1A1	 (2)	
	Total		**10**	
7(a)	Use of similar triangles or algebra	M1		Some progress needed
	Correct relationship established	m1A1		eg $\frac{r-a}{c}=\frac{b-a}{c-d}$
	Hence result convincingly shown	A1	4	AG
(b)(i)	$c=\mathrm{f}(a)=24$, $d=\mathrm{f}(b)=-21$	B1,B1		
	$r=\frac{38}{15}$ (≈ 2.5333)	B1F	3	Allow AWRT 2.53; ft small error
(ii)	$\beta=20^{\frac{1}{3}}\approx 2.714(4)$	M1A1		Allow AWRT 2.71
	So $\beta-r\approx 0.181\approx 0.18$ (AG)	A1	3	Allow only 2dp if earlier values to 3dp
	Total		**10**	

MFP1 (cont)

Q	Solution	Marks	Total	Comments
8(a)	$\int x^{-\frac{3}{4}}\,\mathrm{d}x = 4x^{\frac{1}{4}}\ (+\,c)$	M1A1		M1 if index correct
	This tends to ∞ as $x \to \infty$, so no value	A1F	3	ft wrong coefficient
(b)	$\int x^{-\frac{5}{4}}\,\mathrm{d}x = -4x^{-\frac{1}{4}}\ (+\,c)$	M1A1		M1 if index correct
	$\int_1^{\infty} x^{-\frac{5}{4}}\,\mathrm{d}x = 0 - (-4) = 4$	A1F	3	ft wrong coefficient
(c)	Subtracting 4 leaves ∞, so no value	B1F	1	ft if c has 'no value' in (a) but has a finite answer in (b)
	Total		**7**	
9(a)	Asymptotes are $y = \pm\sqrt{2}x$	M1A1	2	M1A0 if correct but not in required form
(b)	Asymptotes correct on sketch	B1F		With gradients steeper than 1; ft from $y = \pm\, mx$ with $m > 1$
	Two branches in roughly correct positions	B1		
	Approaching asymptotes correctly	B1	3	Asymptotes $y = \pm\, mx$ needed here
(c)(i)	Elimination of y	M1		
	Clearing denominator correctly	M1		
	$x^2 - 2cx - (c^2 + 2) = 0$	m1A1	4	Convincingly found (AG)
(ii)	Discriminant = $8c^2 + 8$	B1		Accept unsimplified
	… > 0 for all c, hence result	E1	2	OE
(iii)	Solving gives $x = c \pm\sqrt{2(c^2+1)}$	M1A1		
	$y = x + c = 2c \pm\sqrt{2(c^2+1)}$	A1	3	Accept $y = c + \dfrac{2c \pm\sqrt{8c^2+8}}{2}$
	Total		**14**	
	TOTAL		**75**	

Further pure 1 - AQA - January 2009

Question 1:

Let's call $f(x)=\sqrt{1+x^2}$

we use the Euler formula: $y_{n+1}=y_n+hf(x_n) \qquad with \;\; h=0.2$

$x_1=0,\; y_1=1 \;,\; f(x_1)=1 \qquad so \;\; y_2=1+0.2\times 1=1.2$

$x_2=0.2,\; y_2=1.2 \;,\; f(x_2)=\sqrt{1+0.2^2}=1.0198$

$$so \;\; y_3=1.2+0.2\times 1.0198$$

$$y_3=1.40396$$

The great majority of candidates were able to make a good start to the paper by giving a correct numerical solution to the differential equation. As on past papers, some candidates used the value of the derivative at the upper end of the interval rather than the lower end. This was perfectly acceptable for full marks, though these candidates gave themselves slightly more calculation to carry out as they were not using the very simple value of the derivative at *x* = 0. Some candidates carried out three iterations instead of two; these candidates lost a mark as well as giving themselves extra work.

Question 2:

$x^2+bx+c=0 \quad has \quad root \;\; 2+3i$

$a)$ The other root is the conjugate $\;2-3i$

$b)\; b=-((2+3i)+(2-3i))= \qquad b=-4$

$c=(2+3i)(2-3i)=4+9=13 \qquad c=13$

Almost every candidate gave the right answer to part (a), and many went on to use the relationships between the roots and coefficients of a quadratic equation to answer part (b) correctly, though quite a number did not insert a minus sign for the coefficient of *x*. Some candidates followed other approaches, such as substituting the known root into the equation, expanding and equating real and imaginary parts, which if done carefully usually led to the right answers.

Question 3:

$$Tan\left(\frac{\pi}{2}-3x\right)=\sqrt{3}=Tan\frac{\pi}{3}$$

$$\frac{\pi}{2}-3x=\frac{\pi}{3}+k\pi$$

$$-3x=\frac{-\pi}{6}+k\pi$$

$$x=\frac{\pi}{18}-k\frac{\pi}{3} \qquad k\in\mathbb{Z}$$

The tangent function is the most 'friendly' trigonometrical function for this type of equation, and it was noticeable that the great majority of candidates were aware that the period of the function was π, not 2π. Unfortunately a substantial number of candidates, after introducing the general term *nπ*, failed to divide it by 3 and thus lost three of the five marks available.

Question 4:

$$S_n=\sum_{r=1}^{n}3r^2-3r+1=3\sum_{r=1}^{n}r^2-3\sum_{r=1}^{n}r+\sum_{r=1}^{n}1$$

$$=3\times\frac{1}{6}n(n+1)(2n+1)-3\times\frac{1}{2}n(n+1)+n$$

$$=\frac{1}{2}n\left[(n+1)(2n+1)-3(n+1)+2\right]$$

$$=\frac{1}{2}n\left(2n^2+n+2n+1-3n-3+2\right)$$

$$S_n=\frac{1}{2}n\left(2n^2\right)=n^3$$

$$b)\;\sum_{r=n+1}^{2n}(3r^2-3r+1)=\sum_{r=1}^{2n}(3r^2-3r+1)-\sum_{r=1}^{n}(3r^2-3r+1)$$

$$=(2n)^3-n^3=7n^3$$

Part (a) of this question was very well answered. The algebra needed was not quite as heavy as is sometimes seen in questions on this topic, and the fact that the answer was given seemed to steer candidates smoothly to a correct solution. Only a small proportion of candidates wrote 1 for Σ1. In part (b), only a minority of candidates showed any awareness of the need for a subtraction, but those who did show this awareness usually found the right answer with very little difficulty.

Question 5:

$$A=\begin{bmatrix} k & k \\ k & -k \end{bmatrix} \quad and \quad B=\begin{bmatrix} -k & k \\ k & k \end{bmatrix}$$

The first nine marks in this question seemed to be found very easy for most candidates to earn. The presence of so many *k*'s in the matrices did not prevent them from carrying out all the calculations correctly and confidently, though sometimes the flow of the reasoning in part (b) was not made totally clear. Part (c)(ii), on the other hand, proved too hard for many candidates, who seemed to resort to guesswork for their answers.

$$a)\,i)\; A+B=\begin{bmatrix} 0 & 2k \\ 2k & 0 \end{bmatrix}$$

$$ii)\; A^2=\begin{bmatrix} 2k^2 & 0 \\ 0 & 2k^2 \end{bmatrix}$$

$$b)\,(A+B)^2=\begin{bmatrix} 0 & 2k \\ 2k & 0 \end{bmatrix}\begin{bmatrix} 0 & 2k \\ 2k & 0 \end{bmatrix}=\begin{bmatrix} 4k^2 & 0 \\ 0 & 4k^2 \end{bmatrix}$$

$$and \quad A^2+B^2=\begin{bmatrix} 2k^2 & 0 \\ 0 & 2k^2 \end{bmatrix}+\begin{bmatrix} 2k^2 & 0 \\ 0 & 2k^2 \end{bmatrix}=\begin{bmatrix} 4k^2 & 0 \\ 0 & 4k^2 \end{bmatrix}$$

$$so\ indeed\ (A+B)^2=A^2+B^2$$

$$c)\,i)\; k=1,\quad A^2=\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

represents an enlargement centre O, scale factor 2

$$ii)\; A=\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

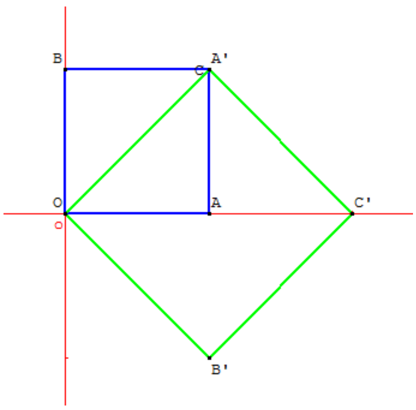
The point A(1,0) is transformed into A'(1,1)

$\text{OA}=1 \;\; and \;\; OA'=\sqrt{2}$

the scale factor of the enlargement is $\sqrt{2}$

The mirror line is the bissector of the angle (AOA'),

the equation of this line is $y=Tan(22.5^o)x$



Question 6:

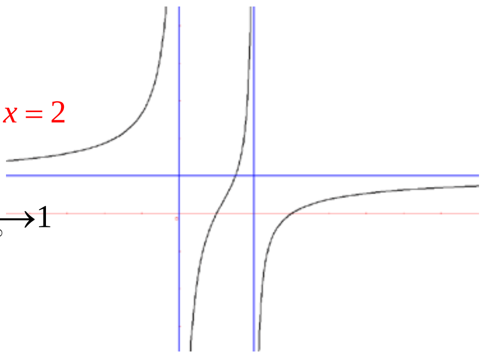
$$y=\frac{(x-1)(x-3)}{x(x-2)}$$

The first two parts of this question were generally well answered, and there were many good sketches in part (a)(iii), though the middle branch of the curve was often badly drawn, many candidates appearing to ignore the helpful information provided about there being no stationary points. Part (b) could be answered very easily by looking at the parts of the graph which were below the *x*-axis, and reading off the two intervals in which this was the case. Instead, many candidates used a purely algebraic approach, and in most cases the inequality was rapidly reduced to the incorrect form $(x-1)(x-3)<0$, though some candidates obtained the correct version, $x(x-1)(x-2)(x-3)<0$, and after making a table of signs came up with the correct solution.

$a)\,i)$ "*vertical asymptotes*" $x=0 \;\; and \;\; x=2$

$$y=\frac{x^2-4x+3}{x^2-2x}=\frac{1-\frac{4}{x}+\frac{3}{x^2}}{1-\frac{2}{x}}\xrightarrow[x\to\infty]{}1$$

$y=1$ *is asymptote to the curve*



$ii)$ The curve intersects the x-axis when $y=0$

which means $(x-1)(x-3)=0 \qquad x=1 \;or\; x=3$

The curve crosses the x-axis at (1,0) and (3,0)

$iii)$

$c)\; y<0$ when the curve is below the x-axis

this happens when $0<x<1 \;\; or \;\; 2<x<3$

Question 7:

$a)$ Let's work out the equation of the line PQ

the gradient is $\dfrac{d-c}{b-a}$

$$Eq: y-c=\frac{d-c}{b-a}(x-a)$$

This line intersects the x-axis when $y=0$

$$so\ \ -c=\frac{d-c}{b-a}(r-a) \qquad -c\left(\frac{b-a}{d-c}\right)=r-a$$

$$r=a+c\left(\frac{b-a}{c-d}\right)$$

$b)\ a=2, b=3\ and\ \ f(x)=20x-x^4$

$$i)\ c=f(a)=20\times2-2^4=40-16=24$$

$$d=f(b)=20\times3-3^4=60-81=-21$$

$$so\ \ r=2+24\left(\frac{3-2}{24+21}\right)=\frac{38}{15}$$

$$ii)\ Solve\ \ f(x)=0 \qquad 20x-x^4=0$$

$$x(20-x^3)=0$$

$$x=0\ \ or\ \ x=\sqrt[3]{20}$$

$$so\ \ \beta-r=\sqrt[3]{20}-\frac{38}{15}\approx 0.18$$

There was a poor response to this question, particularly in part (a) where many candidates seemed to be unfamiliar with the background to the process of linear interpolation. Those who were completely or partially successful followed one of two approaches, a geometrical approach based on similar triangles or an algebraic approach based on the equation of the line *PQ*. In both approaches there were frequent sign errors which prevented the candidate from legitimately obtaining the required formula, but some candidates produced clear, concise and correct proofs.
Part (b)(i) was more often productive for candidates, some of whom used an alternative correct formula for linear interpolation. In part (b)(ii), many candidates did not realise that they had to solve the equation f(*x*) = 0 to find the value of *β*, and those who did solve the equation sometimes lost accuracy in establishing the required approximate value.

Question 8:

$$a)\int x^{-\frac{3}{4}}dx=4x^{\frac{1}{4}}+c$$

$$x^{\frac{1}{4}}\xrightarrow[x\to\infty]{}\infty$$

The integral has no value.

$$b)\int x^{-\frac{5}{4}}dx=-4x^{-\frac{1}{4}}+c\ when\ x\to\infty, x^{-\frac{1}{4}}\to 0$$

$$so\ \int_1^a x^{-\frac{5}{4}}dx=\left[-4x^{-\frac{1}{4}}\right]_0^a=-4a^{-\frac{1}{4}}-(-4)\xrightarrow[a\to\infty]{}4$$

$$\int_1^\infty x^{-\frac{5}{4}}dx=4$$

$$c)\int x^{-\frac{3}{4}}-x^{-\frac{5}{4}}dx=4x^{\frac{1}{4}}+4x^{-\frac{1}{4}}+c=4x^{\frac{1}{4}}\left(1+x^{-\frac{1}{2}}+cx^{-\frac{1}{4}}\right)\xrightarrow[x\to\infty]{}\infty$$

The integral has no value.

Most candidates were able to integrate the given powers of *x*, and many were able to draw the correct conclusions, though sign errors were common in part (b). In part (c), most of those who had been successful so far were able to complete the task successfully, but it was noticeable that some candidates did not take a hint from the award of only one mark for this part of the question: instead of drawing a quick conclusion from the results already obtained they went through the whole process of integrating, substituting and taking limits, possibly losing valuable time.

Question 9:

$$x^2 - \frac{y^2}{2} = 1 \qquad \frac{x^2}{1^2} - \frac{y^2}{\sqrt{2}^2} = 1$$

$a)\ Asymptotes:\ y = \pm\sqrt{2}x$

$b)$

$c)\,i)\ y = x + c$ intersects the hyperbola

so the x-coordinate of the point of intersection satisfy:

$$x^2 - \frac{(x+c)^2}{2} = 1 \qquad (\times 2)$$

$$2x^2 - x^2 - 2cx - c^2 = 2$$

$$x^2 - 2cx - (c^2 + 2) = 0$$

$ii)$ Let's work out the discriminant: $(-2c)^2 - 4\times 1 \times -(c^2+2)$

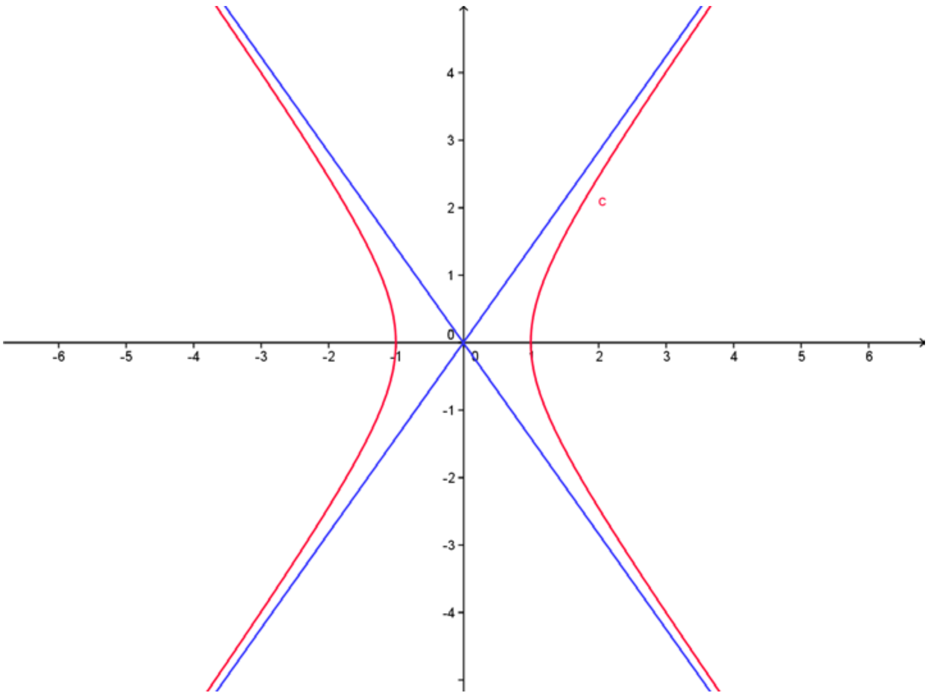
$$4c^2 + 4c^2 + 8 = 8c^2 + 8 > 0$$

The discriminant is positive for all values of c, meaning that the line $y = x + c$ crosses the curve at two points for all the values of c.

$iii)\ Solving\ the\ equation: x = \dfrac{2c \pm \sqrt{8c^2+8}}{2} = \dfrac{2c \pm 2\sqrt{2c^2+2}}{2}$

$$x = c \pm \sqrt{2c^2+2}$$

$y = x + c\ so \qquad y = 2c \pm \sqrt{2c^2+2}$



As in the June 2008 paper, there were many candidates who appeared to struggle with the earlier parts of the final question but then came into their own when they reached the expected test of quadratic theory. Although the asymptotes of a hyperbola had appeared before on MFP1 papers, many candidates seemed unfamiliar with them, and even with the hyperbola itself. A common error in part (a) was to fail to take a square root. In part (b), even those who drew the asymptotes and the two branches in roughly the right positions rarely showed the curve actually approaching the supposed asymptotes.

Part (c)(i) was extremely well answered, the printed answer guiding most candidates unerringly towards a correct piece of algebra, though some made a sign error and then made another one in order to reach the answer. The minus signs caused many candidates to go wrong in finding the discriminant for part (c)(ii), and the fact that the discriminant needed to be strictly positive was not stated in many cases.

The simplest approach to part (c)(iii) was to solve the given equation for *x* in terms of *c* and then to add another *c* to obtain the values of *y*, but most candidates did not seem to realise this. Many candidates obtained a valid quadratic equation for *y* but still failed to write down the quadratic formula as applied to their equation, and thus failed to earn any marks in this part. Those who attempted to substitute values into the formula, either for *x* or for *y*, often omitted the factor *c* from the coefficient of *x* and/or ignored the minus sign in front of this coefficient.

Grade boundaries

Component Code	Component Title	Maximum Scaled Mark	A	B	C	D	E
			Scaled Mark Grade Boundaries				
MFP1	GCE MATHEMATICS UNIT FP1	75	58	51	44	37	31

General Certificate of Education
June 2009
Advanced Subsidiary Examination

MATHEMATICS **MFP1**
Unit Further Pure 1

Monday 1 June 2009 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The equation

$$2x^2 + x - 8 = 0$$

has roots α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) Find the value of $\alpha^2 + \beta^2$. *(2 marks)*

(c) Find a quadratic equation which has roots $4\alpha^2$ and $4\beta^2$. Give your answer in the form $x^2 + px + q = 0$, where p and q are integers. *(3 marks)*

2 A curve has equation

$$y = x^2 - 6x + 5$$

The points A and B on the curve have x-coordinates 2 and $2 + h$ respectively.

(a) Find, in terms of h, the gradient of the line AB, giving your answer in its simplest form. *(5 marks)*

(b) Explain how the result of part (a) can be used to find the gradient of the curve at A. State the value of this gradient. *(3 marks)*

3 The complex number z is defined by

$$z = x + 2\mathrm{i}$$

where x is real.

(a) Find, in terms of x, the real and imaginary parts of:

(i) z^2; *(3 marks)*

(ii) $z^2 + 2z^*$. *(2 marks)*

(b) Show that there is exactly one value of x for which $z^2 + 2z^*$ is real. *(2 marks)*

4 The variables x and y are known to be related by an equation of the form

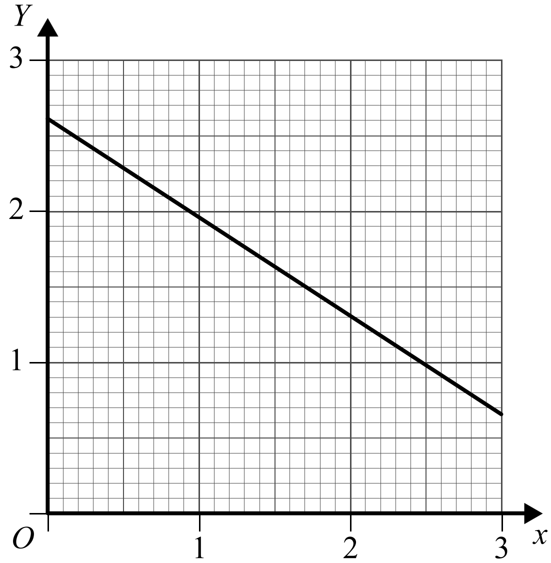
$$y = ab^x$$

where a and b are constants.

(a) Given that $Y = \log_{10} y$, show that x and Y must satisfy an equation of the form

$$Y = mx + c$$ *(3 marks)*

(b) The diagram shows the linear graph which has equation $Y = mx + c$.



Use this graph to calculate:

(i) an approximate value of y when $x = 2.3$, giving your answer to one decimal place;

(ii) an approximate value of x when $y = 80$, giving your answer to one decimal place.

(You are not required to find the values of m and c.) *(4 marks)*

5 (a) Find the general solution of the equation

$$\cos(3x - \pi) = \tfrac{1}{2}$$

giving your answer in terms of π. *(6 marks)*

(b) From your general solution, find all the solutions of the equation which lie between 10π and 11π. *(3 marks)*

6 An ellipse E has equation

$$\frac{x^2}{3} + \frac{y^2}{4} = 1$$

(a) Sketch the ellipse E, showing the coordinates of the points of intersection of the ellipse with the coordinate axes. *(3 marks)*

(b) The ellipse E is stretched with scale factor 2 parallel to the y-axis.

Find and simplify the equation of the curve after the stretch. *(3 marks)*

(c) The **original** ellipse, E, is translated by the vector $\begin{bmatrix} a \\ b \end{bmatrix}$. The equation of the translated ellipse is

$$4x^2 + 3y^2 - 8x + 6y = 5$$

Find the values of a and b. *(5 marks)*

7 (a) Using surd forms where appropriate, find the matrix which represents:

(i) a rotation about the origin through 30° anticlockwise; *(2 marks)*

(ii) a reflection in the line $y = \frac{1}{\sqrt{3}}x$. *(2 marks)*

(b) The matrix **A**, where

$$\mathbf{A} = \begin{bmatrix} 1 & \sqrt{3} \\ \sqrt{3} & -1 \end{bmatrix}$$

represents a combination of an enlargement and a reflection. Find the scale factor of the enlargement and the equation of the mirror line of the reflection. *(2 marks)*

(c) The transformation represented by **A** is followed by the transformation represented by **B**, where

$$\mathbf{B} = \begin{bmatrix} \sqrt{3} & -1 \\ 1 & \sqrt{3} \end{bmatrix}$$

Find the matrix of the combined transformation and give a full geometrical description of this combined transformation. *(5 marks)*

8 A curve has equation

$$y = \frac{x^2}{(x-1)(x-5)}$$

(a) Write down the equations of the three asymptotes to the curve. *(3 marks)*

(b) Show that the curve has no point of intersection with the line $y = -1$. *(3 marks)*

(c) (i) Show that, if the curve intersects the line $y = k$, then the x-coordinates of the points of intersection must satisfy the equation

$$(k-1)x^2 - 6kx + 5k = 0$$

(2 marks)

(ii) Show that, if this equation has equal roots, then

$$k(4k+5) = 0$$

(2 marks)

(d) Hence find the coordinates of the two stationary points on the curve. *(5 marks)*

END OF QUESTIONS

MFP1

Q	Solution	Marks	Totals	Comments
1(a)	$\alpha+\beta=-\frac{1}{2}$, $\alpha\beta=-4$	B1B1	2	
(b)	$\alpha^2+\beta^2=(-\frac{1}{2})^2-2(-4)=8\frac{1}{4}$	M1A1F	2	M1 for substituting in correct formula; ft wrong answer(s) in (a)
(c)	Sum of roots = $4(8\frac{1}{4})=33$	B1F		ft wrong answer in (b)
	Product = $16(\alpha\beta)^2=256$	B1F		ft wrong answer in (a)
	Equation is $x^2-33x+256=0$	B1F	3	ft wrong sum and/or product; allow '$p=-33$, $q=256$'; condone omission of '= 0'
	Total		**7**	
2(a)	When $x=2$, $y=-3$	B1		PI
	Use of $(2+h)^2=4+4h+h^2$	M1		
	Correct method for gradient	M1		
	Gradient = $\dfrac{-3-2h+h^2+3}{h}=-2+h$	A2,1	5	A1 if only one small error made
(b)	As h tends to 0,	E2,1		E1 for '$h=0$'
	... the gradient tends to -2	B1F	3	dependent on at least E1; ft small error in (a)
	Total		**8**	
3(a)(i)	$z^2=(x^2-4)+\mathrm{i}(4x)$	M1A1		M1 for use of $\mathrm{i}^2=-1$
	R and I parts clearly indicated	A1F	3	Condone inclusion of i in I part; ft one numerical error
(ii)	$z^2+2z^*=\left(x^2+2x-4\right)+\mathrm{i}(4x-4)$	M1A1F	2	M1 for correct use of conjugate; ft numerical error in (i)
(b)	z^2+2z^* real if imaginary part zero	M1		
	... ie if $x=1$	A1F	2	ft provided imaginary part linear
	Total		**7**	
4(a)	$\lg(ab^x)=\lg a+\lg(b^x)$	M1		Use of one log law
	... $=\lg a+x\lg b$	M1		Use of another log law
	Correct relationship established	A1	3	
	[SC After M0M0, B2 for correct form]			
(b)(i)	When $x=2.3$, $Y\approx 1.1$, so $y\approx 12.6$	M1A1		Allow 12.7; allow NMS
(ii)	When $y=80$, $Y\approx 1.90$, so $x\approx 1.1$	M1A1	4	M1 for $Y\approx 1.9$, allow NMS
	Total		**7**	

MFP1 (cont)

Q	Solution	Marks	Totals	Comments
5(a)	$\cos\frac{\pi}{3} = \frac{1}{2}$	B1		Decimals/degrees penalised at 6th mark only
	Appropriate use of $\pm$	B1		OE
	Introduction of $2n\pi$	M1		(or $n\pi$) at any stage
	Going from $3x - \pi$ to x	m1		including dividing all terms by 3
	$x = \frac{\pi}{3} \pm \frac{\pi}{9} + \frac{2}{3}n\pi$	A2,1F	6	OE; A1 with decimals and/or degrees; ft wrong first solution
(b)	At least one value in given range	M1		compatible with c's GS
	Correct values $\frac{92}{9}\pi,\ \frac{94}{9}\pi,\ \frac{98}{9}\pi$	A2,1	3	A1 if one omitted or wrong values included; A0 if only one correct value given
	Total		**9**	
6(a)	Ellipse with centre of origin	B1		Allow unequal scales on axes
	$(\pm\sqrt{3}, 0)$ and (0 ± 2) shown on diagram	B2,1	3	Condone AWRT 1.7 for $\sqrt{3}$; B1 for incomplete attempt
(b)	y replaced by $\frac{1}{2}y$	M1A1		M1A0 for $2y$ instead of $\frac{1}{2}y$
	Equation is now $\dfrac{x^2}{3} + \dfrac{y^2}{16} = 1$	A1	3	
(c)	Attempt at completing the square	M1		
	$4(x-1)^2 + 3(y+1)^2 \ldots$	A1A1		
	[Alt: replace x by $x - a$ and y by $y - b$	(M1)		M1 if one replacement correct
	$4x^2 - 8ax + 3y^2 - 6by \ldots$]	(m1A1)		Condone errors in constant terms
	$a = 1$ and $b = -1$	A1A1	5	
	Total		**11**	

MFP1 (cont)

Q	Solution	Marks	Totals	Comments
7(a)(i)	Matrix is $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$	M1A1	2	M1 for $\begin{bmatrix} \cos 30° & \sin 30° \\ -\sin 30° & \cos 30° \end{bmatrix}$ (PI)
(ii)	Matrix is $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$	M1A1	2	M1 for $\begin{bmatrix} \cos 60° & \sin 60° \\ \sin 60° & -\cos 60° \end{bmatrix}$ (PI)
(b)	SF 2, line $y = \frac{1}{\sqrt{3}}x$	B1B1	2	OE
(c)	Attempt at **BA** or **AB**	M1		
	BA = $\begin{bmatrix} 0 & 4 \\ 4 & 0 \end{bmatrix}$	m1A1		m1 if zeros in correct positions
	Enlargement SF 4	B1F		ft use of **AB** (answer still 4) or after **BA** = $\begin{bmatrix} 0 & k \\ k & 0 \end{bmatrix}$
	... and reflection in line $y = x$	B1F	5	ft only from **BA** = $\begin{bmatrix} 0 & k \\ k & 0 \end{bmatrix}$
	Total		**11**	
8(a)	Asymptotes $x = 1$, $x = 5$, $y = 1$	B1 × 3	3	
(b)	$y = -1 \Rightarrow (x-1)(x-5) = -x^2$	M1		OE
	... $\Rightarrow 2x^2 - 6x + 5 = 0$	m1		OE
	Disc't = 36 − 40 < 0, so no pt of int'n	A1	3	convincingly shown (AG)
(c)(i)	$y = k \Rightarrow x^2 = k(x^2 - 6x + 5)$	M1		OE
	... $\Rightarrow (k-1)x^2 - 6kx + 5k = 0$	A1	2	convincingly shown (AG)
(ii)	Discriminant = $36k^2 - 20k(k - 1)$	M1		OE
	... = 0 when $k(4k + 5) = 0$	A1	2	convincingly shown (AG)
(d)	$k = 0$ gives $x = 0$, $y = 0$	B1		
	$k = -\frac{5}{4}$ gives $-\frac{9}{4}x^2 + \frac{30}{4}x - \frac{25}{4} = 0$	M1A1		OE
	$(3x - 5)^2 = 0$, so $x = \frac{5}{3}$	A1		
	$y = -\frac{5}{4}$	B1	5	
	Total		**15**	
	TOTAL		**75**	

Further pure 1 - AQA - June 2009

Question 1:

$2x^2 + x - 8 = 0$ *has roots* α *and* β

$a)\ \alpha + \beta = \frac{-1}{2}$ and $\alpha\beta = \frac{-8}{2} = -4$

$b)\ \alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = \left(-\frac{1}{2}\right)^2 - 2\times -4 = \frac{1}{4} + 8 = 8\tfrac{1}{4}$

$c)\ 4\alpha^2 + 4\beta^2 = 4(\alpha^2+\beta^2) = 4\times 8\tfrac{1}{4} = 33$

$4\alpha^2 \times 4\beta^2 = 16(\alpha\beta)^2 = 16\times 16 = 256$

An equation with roots $4\alpha^2$ *and* $4\beta^2$ *is* $x^2 - 33x + 256 = 0$

Most candidates found this a very good introduction to the paper and gained full marks, or very nearly so. Errors in parts (a) and (b) were rare, and were nearly always sign errors rather than those caused by the use of an incorrect procedure.
In part (c) the most common mistake was to have a 4 instead of a 16 in the product of the roots of the required equation. Some candidates failed to see the connection with part (b) when finding the sum of the roots, but more often than not they still found the right value for this sum.

Question 2:

$y = x^2 - 6x + 5$

for $x = 2,\ y = 4 - 12 + 5 = -3$

for $x = 2 + h,\ y = (2+h)^2 - 6(2+h) + 5 = 4 + 4h + h^2 - 12 - 6h + 5$

$h^2 - 2h - 3$

$A(2,-3)$ *and* $B(2+h, h^2-2h-3)$

$a)$ *The gradient of the line AB*: $\frac{(h^2-2h-3)-(-3)}{2+h-2} = \frac{h^2-2h}{h} = h - 2$

$b)$ When h tends to 0, the chord tends to the tangent

and its gradient is -2

Almost all candidates knew how to find the gradient of the line in part (a) of this question, but a distressingly large minority of them made a sign error in subtracting the *y*-values of *A* and *B*, which led to the introduction of an unwanted term -6/h in the answer. In part (b) this should have caused difficulties, but almost invariably the candidates took this term as tending to zero as *h* tended to zero. Those candidates who wrote "*h* = 0" instead of letting *h* tend to zero lost a mark here, but it is pleasing to report that this error was comparatively rare.

Question 3:

$z = x + 2i$

$a)\ i)\ z^2 = (x+2i)^2 = x^2 + 4ix - 4 = (x^2 - 4) + 4ix$

$\mathrm{Re}(z^2) = x^2 - 4$

$\mathrm{Im}(z^2) = 4x$

$ii)\ z^2 + 2z^* = x^2 - 4 + 4ix + 2x - 4i = (x^2 + 2x - 4) + i(4x - 4)$

$\mathrm{Re}(z^2 + 2z^*) = x^2 + 2x - 4$

$\mathrm{Im}(z^2 + 2z^*) = 4x - 4$

$b)\ z^2 + 2z^*$ *is real when* $4x - 4 = 0 \qquad x = 1$

In part (a) (i) of this question most candidates showed some knowledge of complex numbers but failed to display clearly the real and imaginary parts asked for in the question. Part (a) (ii) appeared to cause candidates no trouble at all; but by way of contrast, very few candidates saw what was required in part (b): many equated the real part to zero, while others equated the real and imaginary parts to each other.

Question 4:

$y = ab^x$

$a)\ \log_{10} y = \log_{10}(ab^x)$

$Y = \log_{10} a + \log_{10} b^x$

$Y = x\log_{10} b + \log_{10} a$

$Y = mx + c \qquad m = \log_{10} b \ \ and \ \ c = \log_{10} a$

$b)\ i)\ when\ \ x = 2.3, Y = 1.1 \ \ so \ \ y = 10^{1.1} \approx 12.6$

$ii)\ y = 80,\ Y = \log_{10} 80 \approx 1.90 \ \ so \ \ x \approx 1.1$

Part (a) was very familiar to the majority of candidates – perhaps **too** familiar in many cases as the result lg (ab^x) = lg *a* + *x* lg *b* was quoted rather than properly shown by the use of the laws of logarithms.
Part (b) presented a less familiar type of challenge. It was not necessary or indeed helpful to try to find values for the constants *m* and *c*. All that was needed was to read values directly from the graph and to convert as appropriate between *y* and *Y*.

Question 5:

$$Cos(3x - \pi) = \frac{1}{2} = Cos\frac{\pi}{3}$$

$$3x - \pi = \frac{\pi}{3} + k2\pi \qquad or \qquad 3x - \pi = -\frac{\pi}{3} + k2\pi$$

$$3x = \frac{4\pi}{3} + k2\pi \qquad or \qquad 3x = \frac{2\pi}{3} + k2\pi$$

$$x = \frac{4\pi}{9} + k\frac{2\pi}{3} \qquad or \qquad x = \frac{2\pi}{9} + k\frac{2\pi}{3}$$

$$b)\ 10\pi < x < 11\pi \qquad \frac{90\pi}{9} < x < \frac{99\pi}{9}$$

$$and \ \ x = \frac{4\pi + k6\pi}{9} \qquad k = 15 \qquad x = \frac{94}{9}\pi$$

$$or \ \ x = \frac{2\pi + k4\pi}{9} \qquad k = 23 \qquad x = \frac{94\pi}{2}$$

$$k = 24 \qquad x = \frac{98\pi}{2}$$

Part (a) was a standard type of question on this paper, but as on past papers it was common to see candidates introducing the general term 2*nπ* **after** they had divided both sides of the equation by 3. Another very frequent source of confusion came from an inappropriate use of the ± symbol. Candidates who wrote out the whole equation twice, once with a plus sign and once with a minus sign, at an early stage, usually obtained a correct general solution, whereas those who used the plus-or-minus sign often ran into errors when adding π to each side of the equation.
Part (b) was very poorly answered. Many candidates used *n* = 10 or *n* = 11 in their general solution and did not seem to notice that the resulting values of *x* were well below the minimum value of 10π required by the question. Some candidates wrote down the correct answers but failed to indicate how they had 'found' them 'from their general solutions' as specified in the wording of the question.

Question 6:

$$\frac{x^2}{3} + \frac{y^2}{4} = 1$$

$a)\ When\ \ x = 0,\ \ y = \pm 2$

$When\ \ y = 0,\ \ x = \pm\sqrt{3}$

$b)$ The equation of the ellipse after the transformation is : $\dfrac{x^2}{3} + \dfrac{\left(\dfrac{y}{2}\right)^2}{4} = 1$

$$\frac{x^2}{3} + \frac{y^2}{16} = 1$$

$c)$ The equation of the ellipse after the translation is : $\dfrac{(x-a)^2}{3} + \dfrac{(y-b)^2}{4} = 1$

$$4(x-a)^2 + 3(y-b)^2 = 12$$

$$4x^2 - 8ax + 4a^2 + 3y^2 - 6by + 3b^2 = 12$$

$8a = 8 \quad gives \ \ a = 1$

$-6b = 6 \ \ gives \ \ b = -1$

The sketches of the ellipse in part (a) of this question were mostly satisfactory, though some candidates failed to take square roots when working out the required coordinates.
Part (b) was one of those parts of questions where the absence of any explanation made it hard for the examiners to see what the candidates were trying to do. Some gave the answer in simplified form without any preliminary working. On this occasion the examiners condoned this poor examination technique where it looked as if the candidate had done the right thing. But there were many errors here: some modified the *x* term rather than the *y* term, some multiplied the *y* by 2 instead of dividing it by 2, and many failed to include this 2 in the squaring process when simplifying their equations.
Part (c) was found to be rather hard, but many candidates realised that completing the square was needed and at least made an effort, often failing to see the connection between their correct equations and the required numbers *a* and *b*. The alternative approach of applying the translation from the outset, using the letters *a* and *b*, was about as popular as the completing the square method, but the algebra became too heavy for many candidates and the necessary comparing of terms did not take place.

Question 7:

$$a)\, i)\, \text{rotation about the origin } 30^o \text{ anticlockwise}:$$

$$\begin{bmatrix} \cos 30 & -\sin 30 \\ \sin 30 & \cos 30 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

$$ii)\, \text{reflection in the line } y = \frac{1}{\sqrt{3}}x = (Tan30)x:$$

$$\begin{bmatrix} \cos 60 & \sin 60 \\ \sin 60 & -\cos 60 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$$

$$b)\, A = \begin{bmatrix} 1 & \sqrt{3} \\ \sqrt{3} & -1 \end{bmatrix} = 2\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$$

A represents a combination of an enlargement scale factor 2

and the reflection in the line $y = \frac{1}{\sqrt{3}}x$

$c)$ Let's work out BA: $BA = \begin{bmatrix} 0 & 4 \\ 4 & 0 \end{bmatrix}$

This represents an enlargement scale factor 4

and a reflection in the line $y = x$

In this question part (a) was very well answered in general, candidates being able to choose the correct form of matrix for each type of transformation and to insert the correct forms for the necessary sines and cosines.
In part (b), however, the majority of candidates failed to see the close connection between the given matrix and the one that they had found in part (a) (ii). The answers given often appeared to be wild guesses.
In part (c) many candidates saw the need to multiply the two matrices but often in the wrong order. Those who obtained the correct matrix $\begin{bmatrix} 0 & 4 \\ 4 & 0 \end{bmatrix}$ often confused it with a scalar matrix when giving the geometrical interpretation.

Question 8:

$$y=\frac{x^2}{(x-1)(x-5)}$$

a) "vertical asymptotes" $x=1$ and $x=5$

$$y=\frac{x^2}{x^2-6x+5}=\frac{1}{1-\frac{6}{x}+\frac{5}{x^2}}\xrightarrow[x\to\infty]{}1 \qquad y=1$$

b) Let's solve $\frac{x^2}{x^2-6x+5}=-1$

$$x^2=-x^2+6x-5$$

$$2x^2-6x+5=0$$

Discriminant: $(-6)^2-4\times2\times5=36-40=-4<0$

There is no solution, the line $y=-1$ does not cross the curve

c) *i*) If $y=k$ crosses the curve, then x is solution to $\frac{x^2}{x^2-6x+5}=k$

$$x^2=kx^2-6kx+5k$$

$$(k-1)x^2-6kx+5k=0$$

ii) The equation has equal roots when the discriminant is 0

$$\text{Discriminant} = (-6k)^2-4\times(k-1)\times5k$$

$$=36k^2-20k^2+20k=16k^2+20k=0$$

$$4k^2+5k=0$$

$$k(4k+5)=0$$

d) *This equation gives* $k=0$ *or* $k=-\frac{5}{4}$

for $k=0$, *we have* $-x^2=0$ $\qquad x=0$ *and* $y=0$

for $k=-\frac{5}{4}$, *we have* $-\frac{9}{4}x^2+\frac{30}{4}x-\frac{25}{4}=0$

$$9x^2-30x+25=0$$

$$(3x-5)^2=0 \qquad x=\frac{5}{3} \text{ and } y=-\frac{5}{4}$$

The stationary points are (0,0) and $(\frac{5}{3},-\frac{5}{4})$

Part (a) was usually well answered, with most candidates writing down the equations of the two vertical asymptotes but then struggling to establish the equation of the horizontal asymptote; although the correct answer $y = 1$ appeared more frequently than the most common wrong answer, $y = 0$.
Part (b) was slightly unfamiliar, but most candidates tackled it confidently. A sign error often led to the disappearance of the $x2$ term, while in other cases the quadratic and its discriminant were correctly found but the candidates omitted to mention the essential feature of the discriminant, ie the fact that it was negative, indicating the absence of any points of intersection.
Candidates were mostly on familiar ground with part (c), but marks were sometimes lost in part (c) (i) by a failure to show enough steps to justify the printed answer, and in part (c) (ii) by a sign error which prevented the candidate from reaching the required equation legitimately.
Most candidates were able to make a good attempt at part (d) in the time remaining for them. The origin was usually given correctly as one stationary point, and many found the x-coordinate of the other stationary point after a more or less lengthy calculation. Many failed to see that the y-coordinate was simply the value of k used in this calculation, but they were able to calculate it correctly from the equation of the curve.

Grade boundaries

Component Code	Component Title	Maximum Scaled Mark	Scaled Mark Grade Boundaries A	B	C	D	E
MFP1	MATHEMATICS UNIT MFP1	75	61	53	45	37	29

General Certificate of Education
Advanced Subsidiary Examination
January 2010

Mathematics MFP1

Unit Further Pure 1

Wednesday 13 January 2010 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables
- an insert for use in Question 6 (enclosed).

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The **Examining Body** for this paper is AQA. The **Paper Reference** is MFP1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- Fill in the boxes at the top of the insert.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The quadratic equation

$$3x^2 - 6x + 1 = 0$$

has roots α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) Show that $\alpha^3 + \beta^3 = 6$. *(3 marks)*

(c) Find a quadratic equation, with integer coefficients, which has roots $\dfrac{\alpha^2}{\beta}$ and $\dfrac{\beta^2}{\alpha}$. *(4 marks)*

2 The complex number z is defined by

$$z = 1 + \mathrm{i}$$

(a) Find the value of z^2, giving your answer in its simplest form. *(2 marks)*

(b) Hence show that $z^8 = 16$. *(2 marks)*

(c) Show that $(z^*)^2 = -z^2$. *(2 marks)*

3 Find the general solution of the equation

$$\sin\left(4x + \frac{\pi}{4}\right) = 1$$ *(4 marks)*

4 It is given that

$$\mathbf{A} = \begin{bmatrix} 1 & 4 \\ 3 & 1 \end{bmatrix}$$

and that $\mathbf{I}$ is the 2×2 identity matrix.

(a) Show that $(\mathbf{A} - \mathbf{I})^2 = k\mathbf{I}$ for some integer k. *(3 marks)*

(b) Given further that

$$\mathbf{B} = \begin{bmatrix} 1 & 3 \\ p & 1 \end{bmatrix}$$

find the integer p such that

$$(\mathbf{A} - \mathbf{B})^2 = (\mathbf{A} - \mathbf{I})^2$$ *(4 marks)*

5 (a) Explain why $\int_0^{\frac{1}{16}} x^{-\frac{1}{2}}\,\mathrm{d}x$ is an improper integral. *(1 mark)*

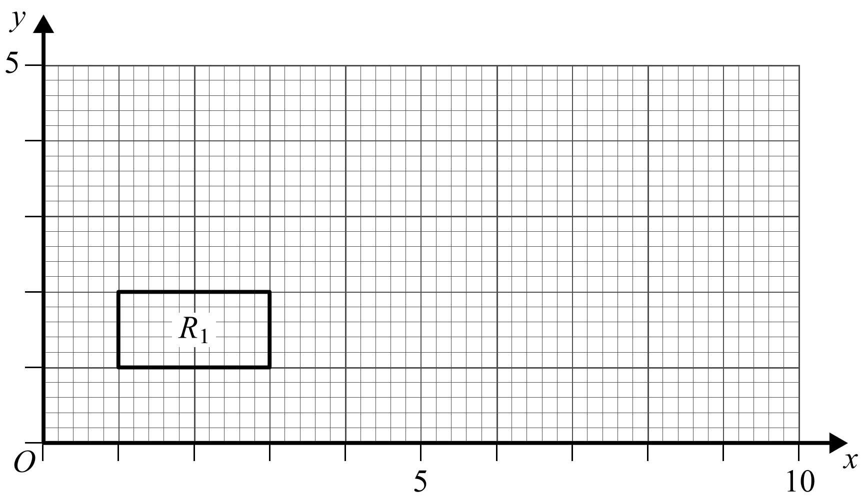
(b) For each of the following improper integrals, find the value of the integral **or** explain briefly why it does not have a value:

(i) $\int_0^{\frac{1}{16}} x^{-\frac{1}{2}}\,\mathrm{d}x$; *(3 marks)*

(ii) $\int_0^{\frac{1}{16}} x^{-\frac{5}{4}}\,\mathrm{d}x$. *(3 marks)*

6 [**Figure 1**, printed on the insert, is provided for use in this question.]

The diagram shows a rectangle R_1 .



(a) The rectangle R_1 is mapped onto a second rectangle, R_2 , by a transformation with matrix $\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$.

(i) Calculate the coordinates of the vertices of the rectangle R_2 . *(2 marks)*

(ii) On **Figure 1**, draw the rectangle R_2 . *(1 mark)*

(b) The rectangle R_2 is rotated through 90° clockwise about the origin to give a third rectangle, R_3 .

(i) On **Figure 1**, draw the rectangle R_3 . *(2 marks)*

(ii) Write down the matrix of the rotation which maps R_2 onto R_3 . *(1 mark)*

(c) Find the matrix of the transformation which maps R_1 onto R_3 . *(2 marks)*

7 A curve C has equation $y = \dfrac{1}{(x-2)^2}$.

(a) (i) Write down the equations of the asymptotes of the curve C. *(2 marks)*

(ii) Sketch the curve C. *(2 marks)*

(b) The line $y = x - 3$ intersects the curve C at a point which has x-coordinate α.

(i) Show that α lies within the interval $3 < x < 4$. *(2 marks)*

(ii) Starting from the interval $3 < x < 4$, use interval bisection twice to obtain an interval of width 0.25 within which α must lie. *(3 marks)*

8 (a) Show that

$$\sum_{r=1}^{n} r^3 + \sum_{r=1}^{n} r$$

can be expressed in the form

$$kn(n+1)(an^2 + bn + c)$$

where k is a rational number and a, b and c are integers. *(4 marks)*

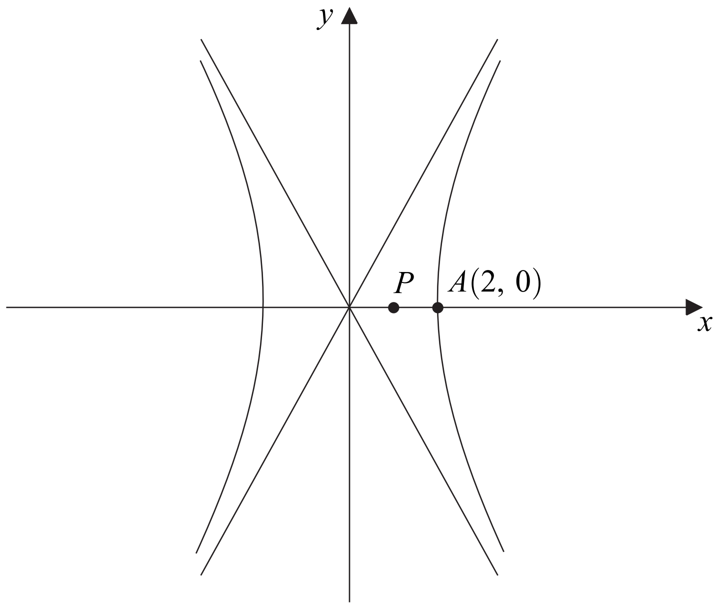
(b) Show that there is exactly one positive integer n for which

$$\sum_{r=1}^{n} r^3 + \sum_{r=1}^{n} r = 8\sum_{r=1}^{n} r^2$$ *(5 marks)*

9 The diagram shows the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

and its asymptotes.



The constants a and b are positive integers.

The point A on the hyperbola has coordinates (2, 0).

The equations of the asymptotes are $y = 2x$ and $y = -2x$.

(a) Show that $a = 2$ and $b = 4$. *(4 marks)*

(b) The point P has coordinates (1, 0). A straight line passes through P and has gradient m. Show that, if this line intersects the hyperbola, the x-coordinates of the points of intersection satisfy the equation

$$(m^2 - 4)x^2 - 2m^2x + (m^2 + 16) = 0$$ *(4 marks)*

(c) Show that this equation has equal roots if $3m^2 = 16$. *(3 marks)*

(d) There are two tangents to the hyperbola which pass through P. Find the coordinates of the points at which these tangents touch the hyperbola.

(No credit will be given for solutions based on differentiation.) *(5 marks)*

END OF QUESTIONS

Q	Solution	Mark	Total	Comments
1(a)	$\alpha+\beta=2,\ \alpha\beta=\frac{1}{3}$	B1B1	2	
(b)	$\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$	M1		or other appropriate formula
	$\ldots=8-3(\frac{1}{3})(2)=6$	m1A1	3	m1 for substn of numerical values; A1 for result shown (AG)
(c)	Sum of roots = $\dfrac{\alpha^3+\beta^3}{\alpha\beta}$	M1		
	$\ldots=\dfrac{6}{\frac{1}{3}}=18$	A1F		ft wrong value for $\alpha\beta$
	Product = $\alpha\beta=\frac{1}{3}$	B1F		ditto
	Equation is $3x^2-54x+1=0$	A1F	4	Integer coeffs and "= 0" needed; ft wrong sum and/or product
	Total		**9**	
2(a)	$z^2=1+2\mathrm{i}+\mathrm{i}^2=2\mathrm{i}$	M1A1	2	M1 for use of $\mathrm{i}^2=-1$
(b)	$z^8=(2\mathrm{i})^4$	M1		or equivalent complete method
	$\ldots=16\mathrm{i}^4=16$	A1	2	convincingly shown (AG)
(c)	$(z^*)^2=(1-\mathrm{i})^2$	M1		for use of $z^*=1-\mathrm{i}$
	$\ldots=-2\mathrm{i}=-z^2$	A1	2	convincingly shown (AG)
	Total		**6**	
3	$\sin\frac{\pi}{2}=1$ stated or used	B1		Deg/dec penalised in 4th mark
	Introduction of $2n\pi$	M1		(or $n\pi$) at any stage
	Going from $4x+\frac{\pi}{4}$ to x	m1		incl division of all terms by 4
	$x=\frac{\pi}{16}+\frac{1}{2}n\pi$	A1	4	or equivalent unsimplified form
	Total		**4**	
4(a)	$\mathbf{I}=\begin{bmatrix}1&0\\0&1\end{bmatrix}$	B1		stated or used at any stage
	Attempt at $(\mathbf{A}-\mathbf{I})^2$	M1		with at most one numerical error
	$(\mathbf{A}-\mathbf{I})^2=\begin{bmatrix}0&4\\3&0\end{bmatrix}\begin{bmatrix}0&4\\3&0\end{bmatrix}=12\mathbf{I}$	A1	3	
(b)	$\mathbf{A}-\mathbf{B}=\begin{bmatrix}0&1\\3-p&0\end{bmatrix}$	B1		
	$(\mathbf{A}-\mathbf{B})^2=\begin{bmatrix}3-p&0\\0&3-p\end{bmatrix}$	M1A1		M1 A0 if 3 entries correct
	$\ldots=(\mathbf{A}-\mathbf{I})^2$ for $p=-9$	A1F	4	ft wrong value of k
	Total		**7**	

MFP1

Q	Solution	Mark	Total	Comments
5(a)	$x^{-\frac{1}{2}} \to \infty$ as $x \to 0$	E1	1	Condone "$x^{-\frac{1}{2}}$ has no value at $x = 0$"
(b)(i)	$\int x^{-\frac{1}{2}}\ \mathrm{d}x = 2x^{\frac{1}{2}}\ \ (+c)$	M1A1		M1 for correct power of x
	$\int_0^{\frac{1}{16}} x^{-\frac{1}{2}}\ \mathrm{d}x = \frac{1}{2}$	A1F	3	ft wrong coefficient of $x^{\frac{1}{2}}$
(ii)	$\int x^{-\frac{5}{4}}\ \mathrm{d}x = -4x^{-\frac{1}{4}}\ \ (+c)$	M1A1		M1 for correct power of x
	$x^{-\frac{1}{4}} \to \infty$ as $x \to 0$, so no value	E1F	3	ft wrong coefficient of $x^{-\frac{1}{4}}$
	Total		**7**	
6(a)(i)	Coords (3, 2), (9, 2), (9, 4), (3, 4)	M1A1	2	M1 for multn of x by 3 or y by 2 (PI)
(ii)	R_2 shown correctly on insert	B1	1	
(b)(i)	R_3 shown correctly on insert	B2,1F	2	B1 for rectangle with 2 vertices correct; ft if c's R_2 is a rectangle in 1st quad
(ii)	Matrix of rotation is $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$	B1	1	
(c)	Multiplication of matrices	M1		(either way) or other complete method
	Required matrix is $\begin{bmatrix} 0 & 2 \\ -3 & 0 \end{bmatrix}$	A1	2	
	Total		**8**	
7(a)(i)	Asymptotes $x = 2$, $y = 0$	B1B1	2	
(ii)	One correct branch Both branches correct	B1 B1	2	no extra branches; $x = 2$ shown
(b)(i)	f(3) = −1, f(4) = 3 Sign change, so α between 3 and 4	B1 E1	2	where $f(x) = (x-3)(x-2)^2 - 1$; OE
(ii)	f(3.5) considered first f(3.5) > 0 so $3 < \alpha < 3.5$ f(3.25) < 0 so $3.25 < \alpha < 3.5$	M1 A1 A1	3	OE but must consider $x = 3.5$ Some numerical value(s) needed Condone absence of values here
	Total		**9**	

MFP1

Q	Solution	Mark	Total	Comments
8(a)	$\Sigma r^3 + \Sigma r = \frac{1}{4}n^2(n+1)^2 + \frac{1}{2}n(n+1)$	M1		at least one term correct
	Factor n clearly shown	m1		or $n + 1$ clearly shown to be a factor
	$\ldots = \frac{1}{4}n(n+1)(n^2+n+2)$	A1A1	4	OE; A1 for $\frac{1}{4}$, A1 for quadratic
(b)	Valid equation formed	M1		
	Factors n, $n + 1$ removed	m1		
	$3n^2 - 29n - 10 = 0$	A1		OE
	Valid factorisation or solution	m1		of the correct quadratic
	$n = 10$ is the only pos int solution	A1	5	SC 1/2 for $n = 10$ after correct quad
	Total		**9**	
9(a)	$x = 2, y = 0 \Rightarrow \frac{4}{a^2} - 0 = 1$ so $a = 2$	E2,1		E1 for verif'n or incomplete proof
	Asymps $\Rightarrow \pm\frac{b}{a} = \pm 2$ so $b = 2a = 4$	E2,1	4	ditto
(b)	Line is $y - 0 = m(x - 1)$	B1		OE
	Elimination of y	M1		
	$4x^2 - m^2(x^2 - 2x + 1) = 16$	A1		OE (no fractions)
	So $(m^2 - 4)x^2 - 2m^2x + (m^2 + 16) = 0$	A1	4	convincingly shown (AG)
(c)	Discriminant equated to zero	M1		
	$4m^4 - 4m^4 - 64m^2 + 16m^2 + 256 = 0$	A1		OE
	$-3m^2 + 16 = 0$, hence result	A1	3	convincingly shown (AG)
(d)	$m^2 = \frac{16}{3} \Rightarrow \frac{4}{3}x^2 - \frac{32}{3}x + \frac{64}{3} = 0$	M1		
	$x^2 - 8x + 16 = 0$, so $x = 4$	m1A1		
	Method for y-coordinates	m1		using $m = \pm\frac{4}{\sqrt{3}}$ or from equation of hyperbola; dep't on previous m1
	$y = \pm 4\sqrt{3}$	A1	5	
	Total		**16**	
	TOTAL		**75**	

Further pure 1 - AQA - January 2010

Question 1:

$3x^2 - 6x + 1 = 0$ *has roots* α *and* β

$a)\ \alpha + \beta = \frac{6}{3} = 2$ *and* $\alpha\beta = \frac{1}{3}$

$b)\ \alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta) = (2)^3 - 3\times\frac{1}{3}\times 2 = 8 - 2 = 6$

$\alpha^3 + \beta^3 = 6$

$c)\ \frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3+\beta^3}{\alpha\beta} = \frac{6}{\frac{1}{3}} = 18$ *and* $\frac{\alpha^2}{\beta}\times\frac{\beta^2}{\alpha} = \alpha\beta = \frac{1}{3}$

An equation with the roots $\frac{\alpha^2}{\beta}$ *and* $\frac{\beta^2}{\alpha}$ *is* $x^2 - 18x + \frac{1}{3} = 0$

$3x^2 - 54x + 1 = 0$

Most candidates seemed to be very familiar with the techniques needed in this question. The formula for the sum of the cubes of the roots was either quoted confidently and correctly or worked out from first principles. Errors occurred mainly in part (c): algebraic errors in finding the sum or even the product of the roots of the required equation; errors in choosing which numerical values to substitute for αβ or α + β ; and, very frequently, a failure to present the final answer in an acceptable form, with integer coefficients and the "= 0" at the end.

Question 2:

$z = 1 + i$

$a)\ z^2 = (1+i)^2 = 1 + 2i + i^2 = 2i$

$b)\ z^8 = \left(z^2\right)^4 = \left(2i\right)^4 = 16i^4 = 16$

$c)\ (z^*)^2 = (1-i)^2 = 1 - 2i - 1 = -2i = -z^2$

Full marks were usually awarded in this question. Answers to part (b) were sometimes very laborious but eventually correct, but by contrast some answers were so brief as to be not totally convincing, earning one mark out of two. A few candidates fell short of full credit in part (c) by working on $(z^*)^2$ but not mentioning $-z^2$.

Question 3:

$Sin\left(4x + \frac{\pi}{4}\right) = 1 = Sin\frac{\pi}{2}$

$4x + \frac{\pi}{4} = \frac{\pi}{2} + k2\pi$

$4x = \frac{\pi}{4} + k2\pi$

$x = \frac{\pi}{16} + k\frac{\pi}{2}$

This trigonometric equation was slightly more straightforward than usual, in that there was only one solution of the equation between 0 and 2π. For many candidates, this did not appear to make things simpler at all: they applied a general formula for sin θ = sin α and did not always realise that their two solutions were equivalent. They were not penalised as long as the second solution was correct, but this was not always the case. What was extremely pleasing to see from an examiner's point of view was that the majority of candidates carried out the necessary operations in the right order, so that all the terms, including the $2n\pi$ term, were divided by 4.

Question 4:

$a)\ \left(A - I\right)^2 = \left(\begin{bmatrix} 0 & 4 \\ 3 & 0 \end{bmatrix}\right)^2 = \begin{bmatrix} 12 & 0 \\ 0 & 12 \end{bmatrix} = 12I$

$b)\ (A-B)^2 = \left(\begin{bmatrix} 0 & 1 \\ 3-p & 0 \end{bmatrix}\right)^2 = \begin{bmatrix} 3-p & 0 \\ 0 & 3-p \end{bmatrix} = 12I$

so $p = -9$

As usual on this paper, the work on matrices was very good indeed, with most candidates working out all the steps efficiently. Some tried to expand the expressions $(\mathbf{A} - \mathbf{I})^2$ and $(\mathbf{A} - \mathbf{B})^2$ but almost invariably assumed commutativity of multiplication. A rather silly way to lose a mark was to work correctly to the equation 3 − *p* = 12 and then to solve this equation incorrectly, which happened quite frequently.

Question 5:

$a)\int_0^{16} x^{-\frac{1}{2}}dx$ is an improper because $x^{-\frac{1}{2}}$ is not defined when $x = 0$.

$b)\ i)\int x^{-\frac{1}{2}}dx = 2x^{\frac{1}{2}} + c$ $\qquad \int_0^{16} x^{-\frac{1}{2}}dx = \left[2\sqrt{x}\right]_0^{16} = 2\sqrt{16} - 0 = 8$

$ii)\int x^{-\frac{5}{4}}dx = -4x^{-\frac{1}{4}} + c$ $\qquad$ *when* $x \to 0,\ x^{-\frac{1}{4}} \to \infty$

The integral has no value

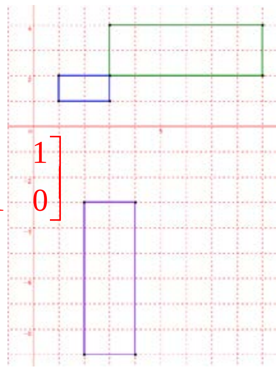
Most candidates showed confidence with the integration needed in this question but were much less confident with the concept of an improper integral. The explanations in part (a) were often very wide of the mark, and indeed quite absurd, while in other cases the statements made were too vague to be worthy of the mark, using the word “it” without making it clear whether this referred to the integrand or to the integrated function. Another mark was often lost at the end of the question, where candidates thought that $0^{-1/4}$ was equal to zero.

Question 6:

$$a)i)\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 3 & 3 \\ 1 & 2 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 3 & 9 & 9 \\ 2 & 4 & 4 & 2 \end{bmatrix}$$

The coordinates of the vertices of R_2 *are* $(3,2), (3,4), (9,4), (9,2)$

$ii)$



$b)i)$

$ii)90^o$ *clockwise is equivalent*

$$\textit{to } 270^o \textit{ anticlockwise}: \begin{bmatrix} \cos 270 & -\sin 270 \\ \sin 270 & \cos 270 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$c)\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \times \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ -3 & 0 \end{bmatrix}$$

As the candidates turned the page to tackle this question, there was a noticeable dip in the level of their performance. The majority of candidates showed a surprising lack of ability to work out the coordinates of image points under a transformation given by a matrix. A common misunderstanding was to carry out a two-way stretch with centre (1, 1) instead of with centre (0, 0). Luckily the candidates still had the chance to carry out the required rotation in part (b)(i) using their rectangle from part (a). Part (b)(ii) was often answered poorly, some candidates being confused by the clockwise rotation, when the formula booklet assumes an anticlockwise rotation, and many candidates failing to give numerical values for cos 270° and sin 270°. Most candidates realised that a matrix multiplication was needed in part (c), but many used the wrong matrices or multiplied the matrices in the wrong

Question 7

$$y = \frac{1}{(x-2)^2}$$

$a)i)$"*Vertical asymptote*" $x = 2$

When $x \to \infty, y \to 0$ $\qquad y = 0$ *is asymptote to the curve.*

$ii)$

$b)$*The line* $y = x - 3$ intersects the curve. x satisfy both equations

$$x - 3 = \frac{1}{(x-2)^2} \qquad (x-3)(x-2)^2 - 1 = 0$$

Let's call $f(x) = (x-3)(x-2)^2 - 1$

$f(3) = -1 < 0$ *and* $\qquad f(4) = 3 > 0$

According the the change sign rule,

we know that there is a least one solution between 3 and 4.

$ii)\ f(3.5) = 0.125 > 0 \qquad 3 < \alpha < 3.5$

$f(3.25) = -0.609375 < 0 \qquad 3.25 < \alpha < 3.5$

Most candidates started well by writing down $x = 2$ as the equation of one asymptote to the given curve, and then struggled to find the horizontal asymptote, though most were ultimately successful. The graph was often drawn correctly but almost equally often it appeared with one of its branches below the x-axis. In part (b), most candidates went to some trouble establishing a function which would have the value 0, or 1, at the point of intersection. The most popular technique for this was to clear denominators to obtain a cubic in factorised form, often converted unhelpfully into expanded form. Other candidates often used subtraction to obtain a suitable function. Once this was done, the way was clear for a candidate to earn 5 marks, but in some cases only 2 of the 5 were gained as interval bisection was not used as required by the question.

Question 8

$$a)\sum_{r=1}^{n} r^3+\sum_{r=1}^{n} r = \frac{1}{4}n^2(n+1)^2+\frac{1}{2}n(n+1)$$

$$=\frac{1}{4}n(n+1)\left[n(n+1)+2\right]$$

$$=\frac{1}{4}n(n+1)(n^2+n+2)$$

$$b)\,8\sum_{r=1}^{n} r^2 = 8\times\frac{1}{6}n(n+1)(2n+1)=\frac{4}{3}n(n+1)(2n+1)$$

$$therefore,\ \sum_{r=1}^{n} r^3+\sum_{r=1}^{n} r = 8\sum_{r=1}^{n} r^2 \quad when$$

$$\frac{1}{4}(n^2+n+2)=\frac{4}{3}(2n+1) \qquad (\times 12)$$

$$3(n^2+n+2)=16(2n+1)$$

$$3n^2+3n+6=32n+16$$

$$3n^2-29n-10=0$$

$$(3n+1)(n-10)=0$$

$$n=10\ is\ the\ only\ \ positive\ \text{integer}\ solution$$

Candidates who were accustomed to look for common factors — an approach almost always needed in questions on this topic — were able to obtain high marks in both parts, though some of these candidates surprisingly failed to solve the quadratic in part (b). Candidates who preferred to expand and simplify everything and then hope to spot some factors were often successful in part (a) but could not realistically hope for more than one mark in part (b).

Question 9

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

$a)\ A(2,0)\ \textit{belongs to the curve so}\ \frac{4}{a^2}=1 \qquad a^2=4 \qquad a=2$

$\textit{The asymptotes are}\ y=\pm\frac{b}{a}x \qquad \textit{so}\ \frac{b}{a}=2 \qquad b=4$

$b)\ P(1,0)\ \textit{gradient}\ m$

$\textit{The equation of the line is}\ y-0=m(x-1)$

$$y=mx-m$$

This line intersects the parabola so the x-coordinate of the points of intersection satisfies:

$$\frac{x^2}{4}-\frac{(mx-m)^2}{16}=1$$

$$4x^2-m^2x^2+2m^2x-m^2=16$$

$$(4-m^2)x^2+2m^2x-(m^2+16)=0$$

$$(m^2-4)x^2-2m^2x+(m^2+16)=0 \qquad (E)$$

$c)$ The equation has equal roots when the discriminant is 0

$$(-2\mathrm{m}^2)^2-4\times(m^2-4)\times(m^2+16)=0$$

$$4m^4-4m^4-48m^2+256=0$$

$$48m^2=256$$

$$3m^2=16$$

$d)\ 3m^2=16 \quad \textit{gives}\ m^2=\frac{16}{3} \qquad m=\pm\frac{4}{\sqrt{3}}=\pm\frac{4\sqrt{3}}{3}$

$\textit{The equation}\ (E)\ \textit{becomes} \quad \left(\frac{16}{3}-4\right)x^2-2\times\frac{16}{3}x+\left(\frac{16}{3}+16\right)=0$

$$\frac{4}{3}x^2-\frac{32}{3}x+\frac{64}{3}=0$$

$$x^2-8x+16=0$$

$$(x-4)^2=0$$

$$x=4$$

$y=m(x-1) \qquad \textit{so}\ y=\pm\frac{4\sqrt{3}}{3}\times 3=\pm 4\sqrt{3}$

The coordinates of the points are $(4,4\sqrt{3})\ \textit{and}\ (4,-4\sqrt{3})$

Part (a) was found very hard by most candidates. Many failed to use both pieces of information supplied just before part (a), so that they could establish a = 2 or b = 2a but could not hope to complete the two requests. Whether they were attempting one half or both halves of the question, they often wrote down the results they were supposed to be proving, possibly earning some credit for verifying these results, though the reasoning was sometimes very hard to follow.
Part (b) was much more familiar to well-prepared candidates, but marks were often lost either by a failure to form a correct equation for the straight line or by sign errors after the elimination of y. The solutions to parts (c) and (d) were often presented in the reverse order, but full credit was given for all correct working shown. In part (c), many candidates made a good attempt to deal with the discriminant of the quadratic equation printed in part (b), but were careless about indicating that this discriminant should be equal to zero for equal roots. Once again, sign errors often caused a loss of marks. In part (d), the unique value of x was often found correctly by the stronger candidates, but relatively few of these went on to find the values of y, and those who did sometimes did so via a rather roundabout approach.

Grade boundaries

Code	Title	Max. Scaled Mark	A*	A	B	C	D	E
			Scaled Mark Grade Boundaries and A* Conversion Points					
MFP1	GCE MATHEMATICS UNIT FP1	75	-	68	61	54	47	40

General Certificate of Education
Advanced Subsidiary Examination
June 2010

Mathematics MFP1

Unit Further Pure 1

Thursday 27 May 2010 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

1 A curve passes through the point (1, 3) and satisfies the differential equation

$$\frac{dy}{dx} = 1 + x^3$$

Starting at the point (1, 3), use a step-by-step method with a step length of 0.1 to estimate the y-coordinate of the point on the curve for which $x = 1.3$. Give your answer to three decimal places.

(No credit will be given for methods involving integration.) *(6 marks)*

---

2 It is given that $z = x + iy$, where x and y are real numbers.

(a) Find, in terms of x and y, the real and imaginary parts of

$$(1 - 2i)z - z^*$$ *(4 marks)*

(b) Hence find the complex number z such that

$$(1 - 2i)z - z^* = 10(2 + i)$$ *(2 marks)*

---

3 Find the general solution, in degrees, of the equation

$$\cos(5x - 20°) = \cos 40°$$ *(5 marks)*

---

4 The variables x and y are related by an equation of the form

$$y = ax^2 + b$$

where a and b are constants.

The following approximate values of x and y have been found.

x	2	4	6	8
y	6.0	10.5	18.0	28.2

(a) Complete the table on page 3, showing values of X, where $X = x^2$. *(1 mark)*

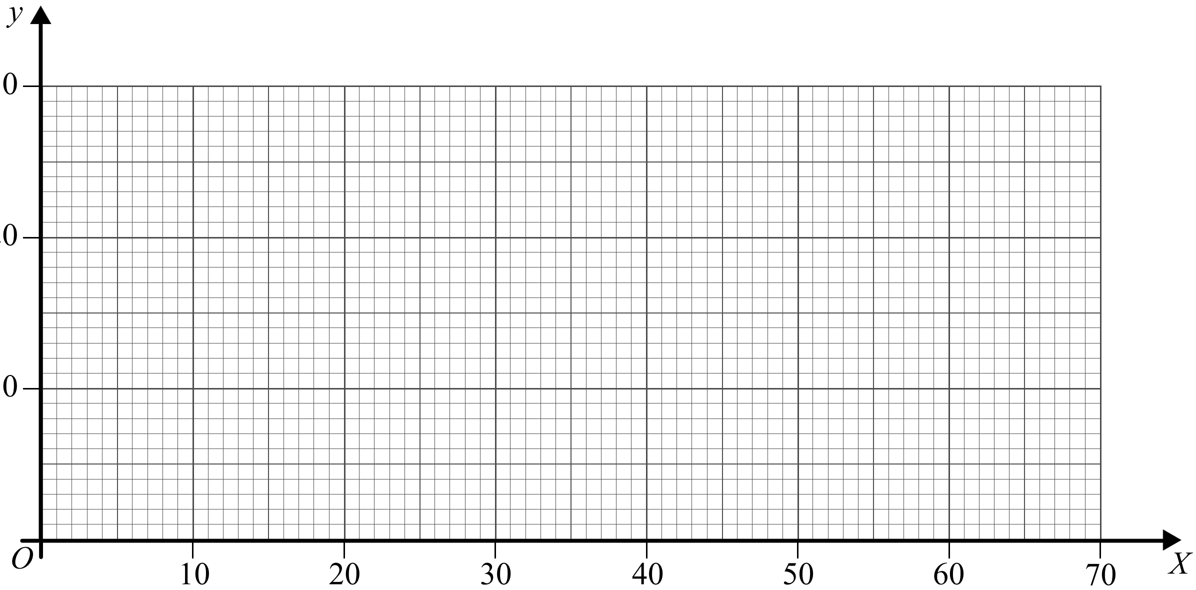
(b) On the diagram on page 3, draw a linear graph relating X and y. *(2 marks)*

(c) Use your graph to find estimates, to two significant figures, for:

(i) the value of x when $y = 15$; *(2 marks)*

(ii) the values of a and b. *(3 marks)*

x	2	4	6	8
X				
y	6.0	10.5	18.0	28.2



5 A curve has equation $y = x^3 - 12x$.

The point A on the curve has coordinates $(2, -16)$.

The point B on the curve has x-coordinate $2 + h$.

(a) Show that the gradient of the line AB is $6h + h^2$. *(4 marks)*

(b) Explain how the result of part **(a)** can be used to show that A is a stationary point on the curve. *(2 marks)*

6 The matrices $\mathbf{A}$ and $\mathbf{B}$ are defined by

$$\mathbf{A} = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

Describe fully the geometrical transformation represented by each of the following matrices:

(a) $\mathbf{A}$; *(2 marks)*

(b) $\mathbf{B}$; *(2 marks)*

(c) $\mathbf{A}^2$; *(2 marks)*

(d) $\mathbf{B}^2$; *(2 marks)*

(e) $\mathbf{AB}$. *(3 marks)*

---

7 (a) (i) Write down the equations of the two asymptotes of the curve $y = \frac{1}{x-3}$. *(2 marks)*

(ii) Sketch the curve $y = \frac{1}{x-3}$, showing the coordinates of any points of intersection with the coordinate axes. *(2 marks)*

(iii) On the same axes, again showing the coordinates of any points of intersection with the coordinate axes, sketch the line $y = 2x - 5$. *(1 mark)*

(b) (i) Solve the equation

$$\frac{1}{x-3} = 2x - 5$$ *(3 marks)*

(ii) Find the solution of the inequality

$$\frac{1}{x-3} < 2x - 5$$ *(2 marks)*

8 The quadratic equation

$$x^2 - 4x + 10 = 0$$

has roots α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) Show that $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{2}{5}$. *(2 marks)*

(c) Find a quadratic equation, with integer coefficients, which has roots $\alpha + \frac{2}{\beta}$ and $\beta + \frac{2}{\alpha}$. *(6 marks)*

---

9 A parabola P has equation $y^2 = x - 2$.

(a) (i) Sketch the parabola P. *(2 marks)*

(ii) On your sketch, draw the two tangents to P which pass through the point $(-2, 0)$. *(2 marks)*

(b) (i) Show that, if the line $y = m(x + 2)$ intersects P, then the x-coordinates of the points of intersection must satisfy the equation

$$m^2x^2 + (4m^2 - 1)x + (4m^2 + 2) = 0$$ *(3 marks)*

(ii) Show that, if this equation has equal roots, then

$$16m^2 = 1$$ *(3 marks)*

(iii) Hence find the coordinates of the points at which the tangents to P from the point $(-2, 0)$ touch the parabola P. *(3 marks)*

END OF QUESTIONS

MFP1

Q	Solution	Marks	Total	Comments
1	First increment is 0.1×2 $(= 0.2)$	M1		variations possible here
	So next value of y is 3.2	A1		PI
	Second inc't is $0.1(1 + 1.1^3) = 0.2331$	m1A1		PI
	Third inc't is $0.1(1 + 1.2^3) = 0.2728$	A1		PI
	So $y \approx 3.7059 \approx 3.706$	A1F	6	ft one numerical error
	Total		**6**	
2(a)	Use of $z^* = x - \mathrm{i}y$	M1		
	Use of $\mathrm{i}^2 = -1$	M1		
	$(1 - 2\mathrm{i})z - z^* = 2y + \mathrm{i}(2y - 2x)$	A2,1	4	A1 if one numerical error made
(b)	$2y = 20$, $2y - 2x = 10$	M1		equate and attempt to solve
	so $z = 5 + 10\mathrm{i}$	A1	2	allow $x = 5$, $y = 10$
	Total		**6**	
3	Introduction of $360n°$	M1		(or $180n°$) at any stage; condone $2n\pi$ (or $n\pi$)
	$5x - 20° = \pm 40°$ $(+360n°)$	B1		OE, eg RHS '40° or 320°'
	Going from $5x - 20°$ to x	m1		including division of all terms by 5
	GS is $x = 4° \pm 8° + 72n°$	A2,1	5	OE; A1 if radians present in answer
	Total		**5**	
4(a)	4, 16, 36, 64 entered in table	B1	1	
(b)	Four points plotted accurately	B1F		ft wrong values in (a)
	Linear graph drawn	B1	2	
(c)(i)	Finding X for $y = 15$ and taking sq root	M1		
	$x \approx 5.3$	A1	2	AWRT 5.2 or 5.3; NMS 1/2
(ii)	Calculation of gradient	M1		
	a = gradient ≈ 0.37	A1		AWRT 0.36 to 0.38; NMS 1/2
	b = y-intercept ≈ 4.5	B1F	3	can be found by calculation; ft c's y-intercept
	Total		**8**	

MFP1 (cont)

Q	Solution	Marks	Total	Comments
5(a)	At B, $y = (2 + h)^3 - 12(2 + h)$	M1		with attempt to expand and simplify
	$= (8 + 12h + 6h^2 + h^3) - (24 + 12h)$ $(= -16 + 6h^2 + h^3)$	B1		correct expansion of $(2 + h)^3$
	Grad $AB = \dfrac{(-16+6h^2+h^3)-(-16)}{(2+h)-2}$	m1		
	$= \dfrac{6h^2+h^3}{h} = 6h + h^2$	A1	4	convincingly shown (AG)
(b)	As $h \to 0$ this gradient $\to 0$ so gradient of curve at A is 0	E2,1	2	E1 for '$h = 0$'
	Total		**6**	
6(a)	Rotation 45° (anticlockwise)(about O)	M1A1	2	M1 for 'rotation'
(b)	Reflection in $y = x \tan 22.5°$	M1A1	2	M1 for 'reflection'
(c)	Rotation 90° (anticlockwise)(about O)	M1A1F	2	M1 for 'rotation' or correct matrix; ft wrong angle in (a)
(d)	Identity transformation	B2,1F	2	ft wrong mirror line in (b); B1 for $\mathbf{B}^2 = \mathbf{I}$
(e)	$\mathbf{AB} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	M1A1		allow M1 if two entries correct
	Reflection in $y = x$	A1	3	
	Total		**11**	
7(a)(i)	Asymptotes $x = 3$ and $y = 0$	B1,B1	2	may appear on graph
(ii)	Complete graph with correct shape	B1		
	Coordinates $\left(0, -\dfrac{1}{3}\right)$ shown	B1	2	
(iii)	Correct line, $(0, -5)$ and $(2.5, 0)$ shown	B1	1	
(b)(i)	$2x^2 - 11x + 14 = 0$	B1		
	$x = 2$ or $x = 3.5$	M1A1	3	M1 for valid method for quadratic
(ii)	$2 < x < 3$, $x > 3.5$	B2,1F	2	B1 for partially correct solution; ft incorrect roots of quadratic (one above 3, one below 3)
	Total		**10**	

MFP1 (cont)

Q	Solution	Marks	Total	Comments
8(a)	$\alpha + \beta = 4$, $\alpha\beta = 10$	B1,B1	2	
(b)	$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta}$	M1		
	$= \frac{4}{10} = \frac{2}{5}$	A1	2	convincingly shown (AG)
(c)	Sum of roots = $(\alpha + \beta) + 2$(ans to (b))	M1		
	$= 4\frac{4}{5}$	A1F		ft wrong value for $\alpha + \beta$
	Product = $\alpha\beta + 4 + \frac{4}{\alpha\beta}$	M1A1		M1 for attempt to expand product (at least two terms correct)
	$= 14\frac{2}{5}$	A1F		ft wrong value for $\alpha\beta$
	Equation is $5x^2 - 24x + 72 = 0$	A1F	6	integer coeffs and '= 0' needed here; ft one numerical error
	Total		**10**	
9(a)(i)	Parabola drawn passing through (2, 0)	M1 A1	 2	with x-axis as line of symmetry
(ii)	Two tangents passing through (−2, 0)	B1B1	2	to c's parabola
(b)(i)	Elimination of y	M1		
	Correct expansion of $(x + 2)^2$	B1		
	Result	A1	3	convincingly shown (AG)
(ii)	Correct discriminant	B1		
	$16m^4 - 8m^2 + 1 = 16m^4 + 8m^2$	M1		OE
	Result	A1	3	convincingly shown (AG)
(iii)	$\frac{1}{16}x^2 - \frac{3}{4}x + \frac{9}{4} = 0$	M1		OE
	$x = 6$, $y = \pm 2$	A1,A1	3	
	Total		**13**	
	TOTAL		**75**	

Further pure 1 - AQA - June 2010

Question 1:

$$Euler\ \ formula: y_{n+1} = y_n + hf(x_n) \qquad with\ h = 0.1\ and\ \ f(x) = 1 + x^3$$

$$x_1 = 1,\ y_1 = 3,\ \ f(x_1) = 2 \qquad so \quad y_2 = 3 + 0.1\times 2 = 3.2$$

$$x_2 = 1.1,\ y_2 = 3.2,\ f(x_2) = 2.331 \qquad so \quad y_3 = 3.2 + 0.1\times 2.331 = 3.4331$$

$$x_3 = 1.2, y_3 = 3.4331,\ f(x_3) = 2.728 \qquad so \quad y_4 = 3.4331 + 0.1\times 2.728 = 3.7059$$

Most candidates scored high marks on this opening question, though there were some who seemed to have no idea what to do. A mark was often lost by carrying out an unwanted fourth iteration. A small number of candidates used the upper boundaries of the intervals rather than the lower boundaries. This was accepted for full marks, but credit was lost if the candidate switched from one method to the other in the course of working through the solution.

Question 2:

$$z = x + iy$$

$$a)(1-2i)z - z^* = (1-2i)(x+iy) - (x-iy)$$

$$= x + iy - 2ix + 2y - x + iy$$

$$= (2y) + i(2y - 2x)$$

$$\text{Re} = 2y$$

$$\text{Im} = 2y - 2x$$

$$b)(1-2i)z - z^* = 10(2+i) = 20 + 10i \ \ when \ \begin{cases} 2y = 20 \\ 2y - 2x = 10 \end{cases} \qquad \begin{cases} y = 10 \\ x = 5 \end{cases}$$

$$z = 5 + 10i$$

The great majority of candidates showed that they had the necessary knowledge of complex numbers to cope with this very straightforward question. In a distressingly high number of instances the work was marred by elementary errors in the algebra, most commonly by a sign error causing $-z^*$ to appear as $-x - iy$. Many candidates also failed to indicate clearly in part (a) which were the real and imaginary parts, though many recovered the mark by using the real and imaginary parts correctly in part (b) of the question.

Question 3:

$$Cos(5x - 20^o) = \cos 40^0$$

$$5x - 20 = 40 + k360 \quad or \quad 5x - 20 = -40 + k360$$

$$5x = 60 + k360 \quad or \quad 5x = -20 + k360$$

$$x = 12^o + k72^o \quad or \quad x = -4^o + k72^o$$

Most candidates introduced a term 360n° into their work at some stage, sometimes at a very late stage indeed, but credit was given for having some awareness of general solutions. A number of candidates gave the equivalent in radians, even though the question specified that degrees were to be used in this case.
Marks were often lost by the omission or misuse of the 'plus-or-minus' symbol. In some cases this was introduced too late, after the candidate had reached the stage of writing '5x = 60°'. In other cases the symbol appeared correctly but then '± 40 + 20' became '± 60'.

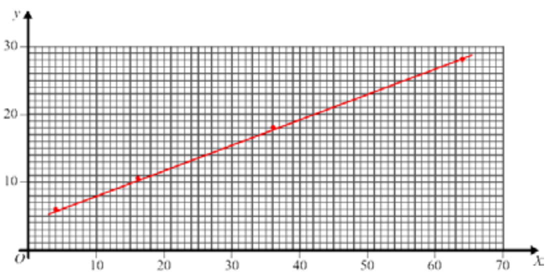
Question 4:

$$y = ax^2 + b$$

a)

x	2	4	6	8
X	4	16	36	64
y	6.0	10.5	18.0	28.2

b)



c) i) When $y = 15$, $X = 28$ $\qquad x \approx 5.3$

$$ii)\ Gradient\ \ a = \frac{28.2 - 6.0}{64 - 4} = 0.37$$

$$b = y - aX = 6.0 - 0.37\times 4 \qquad b \approx 4.5$$

High marks were almost invariably gained in this question. In particular the first three marks were earned by almost all the candidates. Part (c)(i) was often answered without any sign of awareness of a distinction between x and X, a distinction which is of the utmost importance in this type of question. In part (c)(ii) many candidates used calculations based on pairs of coordinates found in the table, but this was accepted as these coordinates could equally have been found from the graph. The value of b often emerged inaccurately from these calculations, though the candidate could so easily have used the y-intercept.

Question 5:

$y = x^3 - 12x$

$A(2,-16)$

$x_B = 2+h, \quad y_B = (2+h)^3 - 12(2+h) = 8 + 6h^2 + 12h + h^3 - 24 - 12h$

$$= h^3 + 6h^2 - 16$$

$B(2+h, h^3 + 12h^2 - 6h - 16)$

$a)$ The gradient of the line $AB = \dfrac{y_B - y_A}{x_B - x_A} = \dfrac{(h^3+6h^2-16)-(-16)}{2+h-2}$

$$= \frac{h^3+6h^2}{h} = 6h + h^2$$

$b)$ when h $\to 0$, the gradient of the line AB tends to the gradient of the tangent at A

$$6h + h^2 \xrightarrow[h\to 0]{} 0$$

The tangent has gradient 0, A is a stationary point.

As has happened in past papers on MFP1, the expansion of the cube of a binomial expression involved some lengthy pieces of algebra for many candidates, though the correct answer was often legitimately obtained. Most candidates were then able to put all the necessary terms into the formula for the gradient of a straight line and obtain the required answer correctly. There was a good response to part (b), where many candidates stated correctly that *h* must **tend** to zero. Only rarely did they say, inappropriately, that it must be *equal* to zero.

Question 6:

$$a)\ A = \begin{bmatrix} Cos\frac{\pi}{4} & -Sin\frac{\pi}{4} \\ Sin\frac{\pi}{4} & Cos\frac{\pi}{4} \end{bmatrix}$$

represents the rotation centre O, $\frac{\pi}{4}\,rad\ (or\ \ 45^o)\ anticlockwise$

$$b)\ B = \begin{bmatrix} Cos\frac{\pi}{4} & Sin\frac{\pi}{4} \\ Sin\frac{\pi}{4} & -Cos\frac{\pi}{4} \end{bmatrix}$$

represent the reflection in the line $y = (Tan\frac{\pi}{8})x$

$c)\ A^2 = A \times A$

represents the rotation centre O, $\frac{\pi}{2}(or\,90^o)$ anticlockwise.

$d)\ B^2 = I \qquad$ identity transformation

$$\text{e)}\ AB = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} Cos\frac{\pi}{2} & -Sin\frac{\pi}{2} \\ Sin\frac{\pi}{2} & Cos\frac{\pi}{2} \end{bmatrix}$$

represents the reflection in the line $y = (Tan\frac{\pi}{4})x$

$$y = x$$

High marks were often earned in this question, generally from the multiplication of the matrices rather than from the geometrical explanations, which tended to be shaky. In parts (c) and (d) the vast majority of candidates calculated a matrix product rather than base their answers purely on the transformations already found in parts (a) and (b). The transformation in part (c) was often given as a reflection rather than a rotation, and in part (d) many candidates stated that the matrix was the identity matrix but did not make any geometrical statement as to what this matrix represented. In part (e) the correct matrix was often obtained but the candidates failed to give the correct geometrical interpretation, or in some cases resorted to a full description of the transformation as a combination of the reflection and rotation found in parts (b) and (a). When this was done correctly, full credit was given.

Question 7:

$a)i)\ y=\dfrac{1}{x-3}$ $\quad x=3$ *is a "vertical" asymptote*

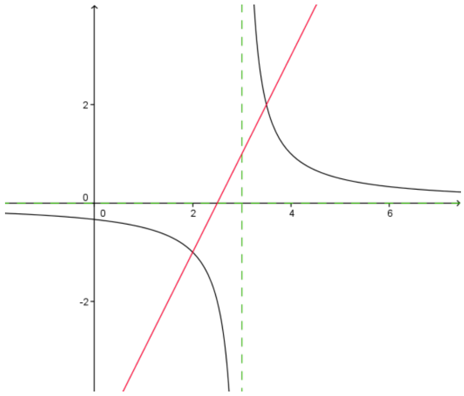
when $x\to\infty, y\to 0,$

$y=0$ *is asymptote to the curve*

$ii)$ *when* $x=0,\ y=-\dfrac{1}{3}$ The curve crosses the y-axis at $(0,-\dfrac{1}{3})$

for all $x, y\neq 0$ The curve does not cross the x-axis

$iii)$



$b)i)\ \dfrac{1}{x-3}=2x-5$ *gives* $1=(2x-5)(x-3)$

$$2x^2-11x+15=1$$
$$2x^2-11x+14=0$$
$$(2x-7)(x-2)=0$$
$$x=3.5\ \text{ or }\ x=2$$

$ii)$ Plot the line $y=2x-5$.

$\dfrac{1}{x-3}<2x-5$: the part of the graph "below the line"

is obtained for $2<x<3$ *and* $x>3.5$

The first eight marks out of the ten available in this question were gained without much difficulty by the majority of candidates, apart from some careless errors such as omitting to indicate the coordinates asked for on the sketch.

By contrast the inequality in part (b)(ii) was badly answered. Few candidates seemed to think of reading off the answers from the graph, the majority preferring an algebraic approach, which if done properly would have been worth much more than the two marks on offer. The algebraic method usually failed at the first step with an illegitimate multiplication of both sides of the inequality by $x-3$. Some candidates multiplied by $(x-3)^2$ but could not cope with the resulting cubic expression.

Question 8:

$x^2-4x+10=0$ *has roots* α *and* β

$a)\ \alpha+\beta=4$ *and* $\alpha\beta=10$

$$b)\ \frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}=\frac{4}{10}=\frac{2}{5}$$

$$c)\ \alpha+\frac{2}{\beta}+\beta+\frac{2}{\alpha}=\alpha+\beta+2(\frac{1}{\alpha}+\frac{1}{\beta})=4+2\times\frac{2}{5}=\frac{24}{5}$$

$$and\ \ (\alpha+\frac{2}{\beta})\times(\beta+\frac{2}{\alpha})=\alpha\beta+2+2+\frac{4}{\alpha\beta}$$

$$=10+4+\frac{4}{10}=\frac{144}{10}=\frac{72}{5}=14\tfrac{2}{5}$$

An equation with these roots is : $x^2-\dfrac{24}{5}x+\dfrac{72}{5}=0$

$$5x^2-24x+72=0$$

This was another well-answered question. The first two parts presented no difficulty to any reasonably competent candidate. In part (c) some candidates, faced with the task of finding the sum of the roots of the required equation, repeated the work done in part (b) rather than quoting the result obtained there. The expansion of the product of the new roots caused some unexpected difficulties, some candidates failing to deal properly with two terms which should have given them constant values. The final mark was often lost by a failure to observe the technical requirements spelt out in the question.

Question 9:

$P: y^2 = x-2$

$a)\ i)$

$\quad ii)$

$b)\ i)\ y = m(x+2)$ intersects P, then the x-*coordinates* of the point of intersection

satisfies both equations: $\begin{cases} y = m(x+2) \\ y^2 = x-2 \end{cases}$

$$\text{this gives}\ \ m^2(x+2)^2 = x-2$$

$$m^2x^2 + 4m^2x + 4m^2 - x + 2 = 0$$

$$m^2x^2 + (4m^2-1)x + (4m^2+2) = 0 \qquad (Eq)$$

$ii)$ this equations has equal roots if the discriminant is 0

$$\left(4m^2-1\right)^2 - 4\times m^2 \times \left(4m^2+2\right) = 0$$

$$16m^4 - 8m^2 + 1 - 16m^4 - 8m^2 = 0$$

$$16m^2 = 1$$

$iii)\ y = m(x+2)$ is the equation of the line going through P with gradient m

This line is a tangent when $16\text{m}^2 = 1$ $\qquad m = \pm\frac{1}{4}$

If $m^2 = \frac{1}{16}$, *the equation* (Eq) *becomes*: $\frac{1}{16}x^2 - \frac{3}{4}x + \frac{9}{4} = 0$

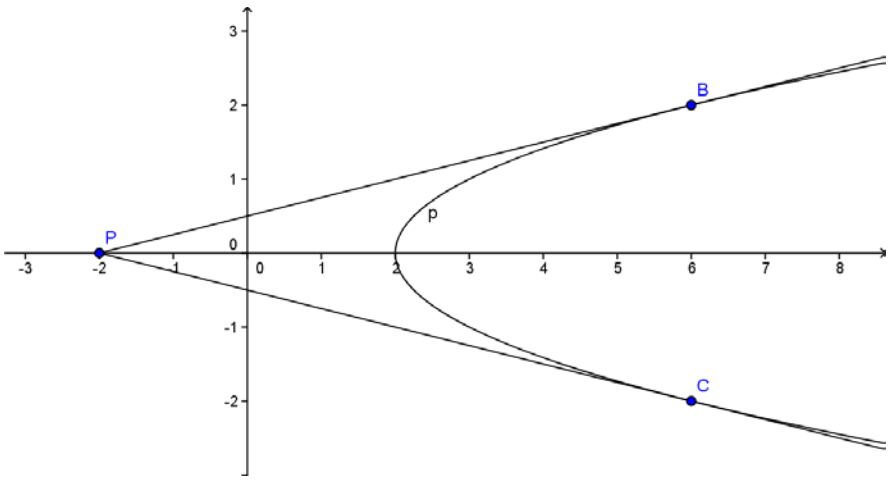
$$x^2 - 12x + 36 = 0$$

$$(x-6)^2 = 0$$

$$x = 6$$

$$\text{then}\ \ y = \pm\frac{1}{4}(x+2) = \pm 2$$

The tangents to the parabola from P touch it at $(6,2)$ *and* $(6,-2)$



The sketch of the parabola *P* was generally well attempted. When asked to sketch two tangents to this parabola, many candidates revealed a poor understanding of the idea of a tangent to a curve.
Part (b) was found familiar by all good candidates, and parts (b)(i) and (b)(ii) yielded full marks provided that a little care was taken to avoid sign errors. Part (b)(iii) was more demanding but many candidates found their way to earning at least some credit, either by substituting the value of m^2 into the quadratic found in part (b)(i) or by some more roundabout method.

Grade boundaries

Component Code	Component Title	Maximum Scaled Mark	A	B	C	D	E
MFP1	GCE MATHEMATICS UNIT FP1	75	61	54	47	40	33

Scaled Mark Grade Boundaries: A–E

General Certificate of Education
Advanced Subsidiary Examination
January 2011

Mathematics MFP1

Unit Further Pure 1

Friday 14 January 2011 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

1 The quadratic equation $x^2 - 6x + 18 = 0$ has roots α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) Find a quadratic equation, with integer coefficients, which has roots α^2 and β^2. *(4 marks)*

(c) Hence find the values of α^2 and β^2. *(1 mark)*

---

2 (a) Find, in terms of p and q, the value of the integral $\displaystyle\int_p^q \frac{2}{x^3}\,\mathrm{d}x$. *(3 marks)*

(b) Show that only one of the following improper integrals has a finite value, and find that value:

(i) $\displaystyle\int_0^2 \frac{2}{x^3}\,\mathrm{d}x$;

(ii) $\displaystyle\int_2^\infty \frac{2}{x^3}\,\mathrm{d}x$. *(3 marks)*

---

3 (a) Write down the 2×2 matrix corresponding to each of the following transformations:

(i) a rotation about the origin through 90° clockwise; *(1 mark)*

(ii) a rotation about the origin through 180°. *(1 mark)*

(b) The matrices $\mathbf{A}$ and $\mathbf{B}$ are defined by

$$\mathbf{A} = \begin{bmatrix} 2 & 4 \\ -1 & -3 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} -2 & 1 \\ -4 & 3 \end{bmatrix}$$

(i) Calculate the matrix $\mathbf{AB}$. *(2 marks)*

(ii) Show that $(\mathbf{A} + \mathbf{B})^2 = k\mathbf{I}$, where $\mathbf{I}$ is the identity matrix, for some integer k. *(3 marks)*

(c) Describe the single geometrical transformation, or combination of two geometrical transformations, represented by each of the following matrices:

(i) $\mathbf{A} + \mathbf{B}$; *(2 marks)*

(ii) $(\mathbf{A} + \mathbf{B})^2$; *(2 marks)*

(iii) $(\mathbf{A} + \mathbf{B})^4$. *(2 marks)*

---

4 Find the general solution of the equation

$$\sin\left(4x - \frac{2\pi}{3}\right) = -\frac{1}{2}$$

giving your answer in terms of π. *(6 marks)*

---

5 (a) It is given that $z_1 = \frac{1}{2} - \mathrm{i}$.

(i) Calculate the value of ${z_1}^2$, giving your answer in the form $a + b\mathrm{i}$. *(2 marks)*

(ii) Hence verify that z_1 is a root of the equation

$$z^2 + z^* + \frac{1}{4} = 0$$ *(2 marks)*

(b) Show that $z_2 = \frac{1}{2} + \mathrm{i}$ also satisfies the equation in part **(a)(ii)**. *(2 marks)*

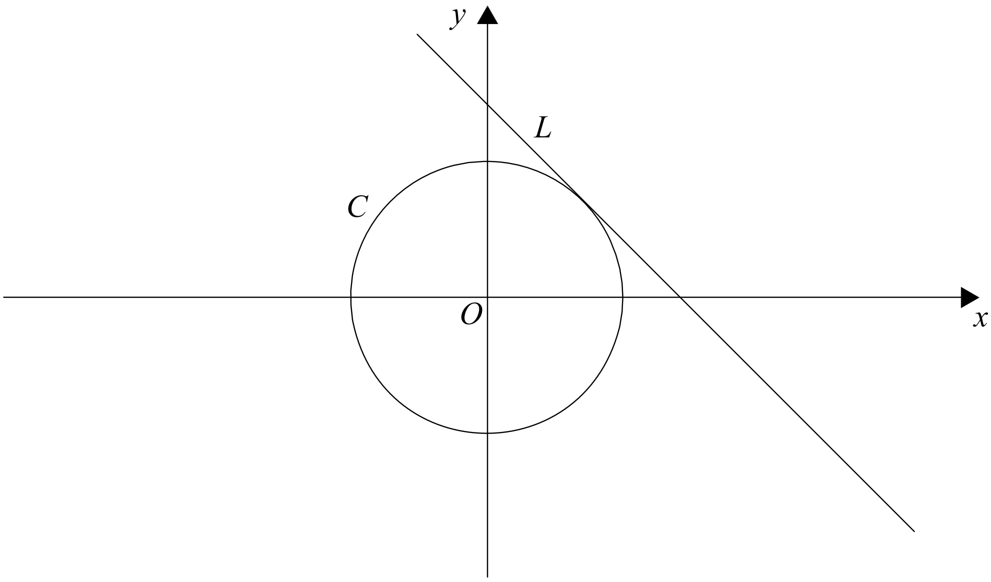
(c) Show that the equation in part **(a)(ii)** has two equal **real** roots. *(2 marks)*

---

6 The diagram shows a circle C and a line L, which is the tangent to C at the point $(1,\ 1)$. The equations of C and L are

$$x^2 + y^2 = 2 \quad \text{and} \quad x + y = 2$$

respectively.



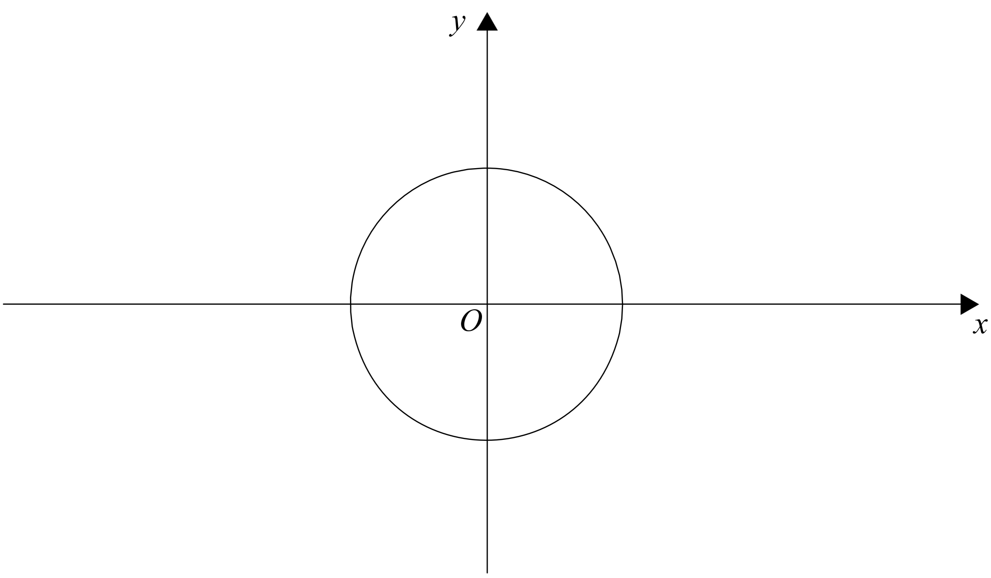
The circle C is now transformed by a stretch with scale factor 2 parallel to the x-axis. The image of C under this stretch is an ellipse E.

(a) **On the diagram below**, sketch the ellipse E, indicating the coordinates of the points where it intersects the coordinate axes. *(4 marks)*

(b) Find equations of:

(i) the ellipse E; *(2 marks)*

(ii) the tangent to E at the point $(2,\ 1)$. *(2 marks)*



7 A graph has equation

$$y = \frac{x-4}{x^2+9}$$

(a) Explain why the graph has no vertical asymptote and give the equation of the horizontal asymptote. *(2 marks)*

(b) Show that, if the line $y = k$ intersects the graph, the x-coordinates of the points of intersection of the line with the graph must satisfy the equation

$$kx^2 - x + (9k+4) = 0$$ *(2 marks)*

(c) Show that this equation has real roots if $-\frac{1}{2} \leqslant k \leqslant \frac{1}{18}$. *(5 marks)*

(d) Hence find the coordinates of the two stationary points on the curve.

(No credit will be given for methods involving differentiation.) *(6 marks)*

---

8 (a) The equation

$$x^3 + 2x^2 + x - 100\,000 = 0$$

has one real root. Taking $x_1 = 50$ as a first approximation to this root, use the Newton-Raphson method to find a second approximation, x_2, to the root. *(3 marks)*

(b) (i) Given that $S_n = \sum_{r=1}^{n} r(3r+1)$, use the formulae for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$ to show that

$$S_n = n(n+1)^2$$ *(5 marks)*

(ii) The lowest integer n for which $S_n > 100\,000$ is denoted by N.

Show that

$$N^3 + 2N^2 + N - 100\,000 > 0$$ *(1 mark)*

(c) Find the value of N, justifying your answer. *(3 marks)*

MFP1

Q	Solution	Marks	Total	Comments
1(a)	$\alpha + \beta = 6$, $\alpha\beta = 18$	B1B1	2	
(b)	Sum of new roots = $6^2 - 2(18) = 0$	M1A1F		ft wrong value(s) in (a)
	Product = $18^2 = 324$	B1F		ditto
	Equation $x^2 + 324 = 0$	A1F	4	'= 0' needed here; ft wrong value(s) for sum/product
(c)	α^2 and β^2 are $\pm 18\mathrm{i}$	B1	1	
	Total		**7**	
2(a)	$\int 2x^{-3}\,\mathrm{d}x = -x^{-2}\ (+\,c)$	M1A1		M1 for correct index
	$\int_p^q 2x^{-3}\,\mathrm{d}x = p^{-2} - q^{-2}$	A1F	3	OE; ft wrong coefficient of x^{-2}
(b)(i)	As $p \to 0$, $p^{-2} \to \infty$, so no value	B1		
(ii)	As $q \to \infty$, $q^{-2} \to 0$, so value is ¼	M1A1F	3	ft wrong coefficient of x^{-2} or reversal of limits
	Total		**6**	
3(a)(i)	$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$	B1	1	
(ii)	$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$	B1	1	
(b)(i)	$\mathbf{AB} = \begin{bmatrix} -20 & 14 \\ 14 & -10 \end{bmatrix}$	M1A1	2	M1A0 if 3 entries correct
(ii)	$\mathbf{A} + \mathbf{B} = \begin{bmatrix} 0 & 5 \\ -5 & 0 \end{bmatrix}$	B1		
	$(\mathbf{A} + \mathbf{B})^2 = \begin{bmatrix} -25 & 0 \\ 0 & -25 \end{bmatrix}$	B1		
	$\ldots = -25\mathbf{I}$	B1F	3	ft if c's $(\mathbf{A} + \mathbf{B})^2$ is of the form $k\mathbf{I}$
(c)(i)	Rot'n 90° clockwise, enlargem't SF 5	B2, 1	2	OE
(ii)	Rotation 180°, enlargement SF 25	B2, 1F	2	Accept 'enlargement SF −25'; ft wrong value of k
(iii)	Enlargement SF 625	B2, 1F	2	B1 for pure enlargement; ft ditto
	Total		**13**	
4	$\sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$	B1		OE; dec/deg penalised at 6th mark
	$\sin\left(-\frac{5\pi}{6}\right) = -\frac{1}{2}$	B1F		OE; ft wrong first value
	Use of $2n\pi$	M1		(or $n\pi$) at any stage
	Going from $4x - \frac{2\pi}{3}$ to x	m1		including division of all terms by 4
	GS $x = \frac{\pi}{8} + \frac{1}{2}n\pi$ or $x = -\frac{\pi}{24} + \frac{1}{2}n\pi$	A1A1	6	OE
	Total		**6**	

MFP1(cont)

Q	Solution	Marks	Total	Comments
5(a)(i)	${z_1}^2 = \frac{1}{4} - \mathrm{i} + \mathrm{i}^2 = -\frac{3}{4} - \mathrm{i}$	M1A1	2	M1 for use of $\mathrm{i}^2 = -1$
(ii)	LHS $= -\frac{3}{4} - \mathrm{i} + \frac{1}{2} + \mathrm{i} + \frac{1}{4} = 0$	M1A1	2	AG; M1 for z^* correct
(b)	LHS $= -\frac{3}{4} + \mathrm{i} + \frac{1}{2} - \mathrm{i} + \frac{1}{4} = 0$	M1A1	2	AG; M1 for ${z_2}^2$ correct
(c)	z real $\Rightarrow z^* = z$ Discr't zero or correct factorisation	M1 A1	 2	Clearly stated AG
	Total		**8**	
6(a)	Sketch of ellipse	M1		centred at origin
	Correct relationship to circle	A1		
	Coords $\left(\pm 2\sqrt{2}, 0\right), \left(0, \pm\sqrt{2}\right)$	B2,1	4	Accept $\sqrt{8}$ for $2\sqrt{2}$; B1 for any 2 of $x = \pm 2\sqrt{2}$, $y = \pm\sqrt{2}$ allow B1 if all correct except for use of decimals (at least one DP)
(b)(i)	Replacing x by $\frac{x}{2}$	M1		or by $2x$
	E is $\left(\frac{x}{2}\right)^2 + y^2 = 2$	A1	2	OE
(ii)	Tangent is $\frac{x}{2} + y = 2$	M1A1	2	M1 for complete valid method
	Total		**8**	
7(a)	Denom never zero, so no vert asymp	E1		
	Horizontal asymptote is $y = 0$	B1	2	
(b)	$x - 4 = k(x^2 + 9)$ Hence result clearly shown	M1 A1	 2	 AG
(c)	Real roots if $b^2 - 4ac \geq 0$ Discriminant $= 1 - 4k(9k + 4)$ **...** $= -(36k^2 + 16k - 1)$ ... $= -(18k - 1)(2k + 1)$ Result (AG) clearly justified	E1 M1 m1 m1 A1	 5	PI (at any stage) m1 for expansion m1 for correct factorisation eg by sketch or sign diagram
(d)	$k = -\frac{1}{2} \Rightarrow -\frac{1}{2}x^2 - x - \frac{1}{2} = 0$	M1A1		or equivalent using $k = \frac{1}{18}$
	... $\Rightarrow (x+1)^2 = 0 \Rightarrow x = -1$	A1		
	$k = \frac{1}{18} \Rightarrow \frac{1}{18}x^2 - x + \frac{9}{2} = 0$	A1		
	... $\Rightarrow (x-9)^2 = 0 \Rightarrow x = 9$	A1		
	SPs are $\left(-1, -\frac{1}{2}\right), \left(9, \frac{1}{18}\right)$	A1	6	correctly paired
	Total		**15**	

MFP1(cont)

Q	Solution	Marks	Total	Comments
8(a)	$x_2 = 50 - \dfrac{50^3 + 2(50^2) + 50 - 100\ 000}{3(50^2) + 4(50) + 1}$	B1 B1		For numerator (PI by value 30050) For denominator (PI by value 7701)
	$x_2 \approx 46.1$	B1	3	Allow AWRT 46.1
8(b)(i)	$\Sigma r(3r + 1) = 3\Sigma r^2 + \Sigma r$	M1		
	$\ldots = 3\left(\frac{1}{6}n\right)(n+1)(2n+1) + \frac{1}{2}n(n+1)$	m1		correct formulae substituted
	$\ldots = \frac{1}{2}n(n+1)(2n+1+1)$	m1m1		m1 for each factor (n and $n + 1$)
	$\ldots = n(n + 1)^2$ convincingly shown	A1	5	AG
(ii)	Correct expansion of $n(n + 1)^2$	B1	1	and conclusion drawn (AG)
(c)	Attempt at value of S_{46} Attempt at value of S_{45} $S_{45} < 100000 < S_{46}$, so $N = 46$	M1 m1 A1	 3	
	Alternative method Root of equation in (a) is 45.8 So lowest integer value is 46	 (B3)		 Allow AWRT 45.7 or 45.8
	Total		**12**	
	TOTAL		**75**	

Further pure 1 - AQA - January 2011

Question 1:

$a)\ x^2-6x+18=0$ *has roots* α *and* β

$a)\ \alpha+\beta=6$ *and* $\alpha\beta=18$

$b)\ \alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=6^2-2\times 18=0$

$\alpha^2\beta^2=(\alpha\beta)^2=18^2=324$

An equation with roots α^2 *and* β^2 is $x^2+324=0$

$c)\ x^2=-324=(18i)^2$ *so* $\alpha^2=18i$ *and* $\beta^2=-18i$

Most candidates started the paper in confident fashion, earning the first six marks with apparent ease, though some lost the sixth mark through failing to write '= 0' to complete their equation in part (b). Relatively few candidates realised that they were being tested on their knowledge of complex numbers in part (c), and even those who did sometimes failed to obtain the one mark on offer by misidentifying the two correct roots of their equation.

Question 2:

$$a)\ \int_p^q \frac{2}{x^3}dx=\int_p^q 2x^{-3}dx=\left[-x^{-2}\right]_p^q=-q^{-2}+p^{-2}=\frac{1}{p^2}-\frac{1}{q^2}$$

$$b)\ i)\ When\ p\to 0,\frac{1}{p^2}\to\infty, so \int_0^2\frac{2}{x^3}dx\ \ has\ no\ value$$

$$ii)\ When\ q\to\infty,\frac{1}{q^2}\to 0\ \ so\ \int_2^{\infty}\frac{2}{x^3}dx=\frac{1}{4}$$

This question again provided a good number of marks for the vast majority of candidates, who showed an adequate knowledge of integration, but in an unexpectedly large number of cases sign errors were made in part (a), causing the letters *p* and *q* to be interchanged. In part (b), most candidates were able to identify correctly which integral had the finite value.

Question 3:

$$a)\ i)\ 90^o\ clockwise=270^o\ anticlockwise:\begin{bmatrix} Cos270 & -Sin270\\ Sin270 & Cos270\end{bmatrix}$$

$$=\begin{bmatrix} 0 & 1\\ -1 & 0\end{bmatrix}$$

$$ii)\ Rotation 180^o:\begin{bmatrix} Cos180 & -Sin180\\ Sin180 & Cos180\end{bmatrix}=\begin{bmatrix} -1 & 0\\ 0 & -1\end{bmatrix}$$

$$b) i)\ AB=\begin{bmatrix} 2 & 4\\ -1 & -3\end{bmatrix}\times\begin{bmatrix} -2 & 1\\ -4 & 3\end{bmatrix}=\begin{bmatrix} 20 & 14\\ 14 & -10\end{bmatrix}$$

$$ii)\ (A+B)^2=\begin{bmatrix} 0 & 5\\ -5 & 0\end{bmatrix}\times\begin{bmatrix} 0 & 5\\ -5 & 0\end{bmatrix}=\begin{bmatrix} -25 & 0\\ 0 & -25\end{bmatrix}=-25I$$

$$c)\ i)\ A+B=\begin{bmatrix} 0 & 5\\ -5 & 0\end{bmatrix}=5\begin{bmatrix} 0 & 1\\ -1 & 0\end{bmatrix}$$

represents the rotation 90^o anticlockwise followed by an enlargement scale factor 5

$$ii)\ (A+B)^2=-25I=25\begin{bmatrix} -1 & 0\\ 0 & -1\end{bmatrix}$$

represents the rotation 180^o followed by an enlargement scale factor 25

$$iii)\ (A+B)^4=-25I\times -25I=625I$$

represents an enlargement scale factor 625

Like the preceding questions, this one produced a good opportunity for most candidates to score high marks. The first two marks were sometimes lost because the candidate failed to provide numerical values for the sines and cosines in their matrices. Also the first matrix was often that of a 90° anticlockwise rotation, rather than a clockwise one as required. Most candidates earned two marks in part (b)(i), relatively few finding **BA** instead of **AB**. Full marks were very common in part (b)(ii). The geometrical interpretations asked for in part (c) were mostly correct, though some floundered somewhat in the first part. In part (c)(ii), the answer 'enlargement with scale-factor −25' was acceptable, and very common.

Question 4:

$$Sin\left(4x-\frac{2\pi}{3}\right)=-\frac{1}{2}=Sin\left(-\frac{\pi}{6}\right)$$

$$so \qquad 4x-\frac{2\pi}{3}=-\frac{\pi}{6}+k2\pi \quad or \quad 4x-\frac{2\pi}{3}=\pi+\frac{\pi}{6}+k2\pi$$

$$4x=\frac{\pi}{2}+k2\pi \quad or \quad 4x=\frac{11\pi}{6}+k2\pi$$

$$x=\frac{\pi}{8}+k\frac{\pi}{2} \quad or \quad x=\frac{11\pi}{24}+k\frac{\pi}{2} \qquad k\in\mathbb{Z}$$

As mentioned above, there was a pleasing improvement in the way candidates approached this trigonometrical equation, in contrast to what has been seen over the years. Marks were lost, however, by a failure in many cases to find a correct second particular solution before the general term was added in. The fact that the sine of the angle in the equation was negative clearly made this task a little harder than usual.

Question 5:

$$z_1=\frac{1}{2}-i$$

$$i)\, z_1^{\,2}=\left(\frac{1}{2}-i\right)^2=\frac{1}{4}-i+i^2=-\frac{3}{4}-i \qquad (i^2=-1)$$

$$ii)\, z^2+z^*+\frac{1}{4}=\left(-\frac{3}{4}-i\right)+\left(\frac{1}{2}+i\right)+\frac{1}{4}=-\frac{2}{4}+\frac{1}{2}=0$$

$$z \text{ is a solution to the equation } z^2+z^*+\frac{1}{4}=0$$

$$b)\, z_2=\frac{1}{2}+i=z^*$$

(we are going to use the property: $(u+v)^*=u^*+v^*$)

$$\left(z^*\right)^2+\left(z^*\right)^*+\frac{1}{4}=\left(z^2\right)^*+\left(z^*\right)^*+\frac{1}{4}=\left(z^2+z^*+\frac{1}{4}\right)^*=0^*=0$$

z^* is also a solution to the equation $z^2+z^*+\frac{1}{4}=0$

$c)$ If z is real then $z^*=z$ and

the equation become $z^2+z+\frac{1}{4}=0$

$$(z+\frac{1}{2})^2=0$$

$$z=-\frac{1}{2} \text{ is a real repeated root}$$

The first six marks in this question were obtained with apparent ease in the great majority of scripts. Part (c) usually produced no further marks as the candidates omitted the star from z^* without explaining why this was legitimate. Many candidates seemed to treat the given equation as a quadratic, despite the fact that two roots had already been found and two more were now being asked for.

Question 6:

$$x^2+y^2=2$$

$$x+y=2$$

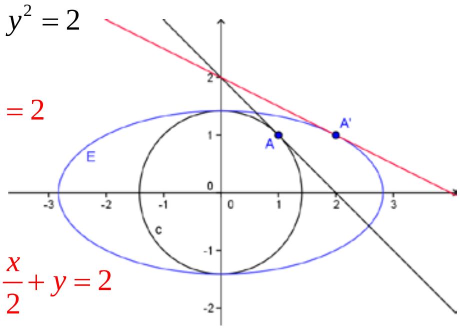
$a)$ The circle crosses the axes at $(0,\sqrt{2}),(0,-\sqrt{2}),(\sqrt{2},0)$, $(-\sqrt{2},0)$

therefore

The ellipse crosses the axes at $(0,\sqrt{2}),(0,-\sqrt{2}),(2\sqrt{2},0)$, $(-2\sqrt{2},0)$

$b)\,i)$ The equation of the ellipse is : $\left(\frac{1}{2}x\right)^2+y^2=2$

$$\frac{x^2}{4}+y^2=2$$



$ii)$ The point of contact and the tangent are affected by the same tranformation

The equation of the new tangent at (2,1) is $\frac{x}{2}+y=2$

In part (a) of this question, most candidates managed to draw an acceptable attempt at an ellipse touching the given circle in the appropriate places. Occasionally the stretch would be applied parallel to the *y*-axis rather than the *x*-axis. The required coordinates were indicated with various levels of accuracy, sometimes appearing as integers, sometimes with minus signs omitted.
In part (b), the most successful candidates were often the ones who wrote the least — all that was needed was to replace *x* by *x/2* in each of the two given equations. Many candidates answered part (b)(i) concisely and correctly but then went into a variety of long methods to find the equation of the tangent in part (b)(ii). Some used implicit differentiation and were usually successful. Others used chain-rule differentiation and usually made errors. Yet others embarked on a very complicated piece of work based on quadratic theory and using a general gradient *m*. This must have consumed a large amount of time and was

Question 7:

$$y=\frac{x-4}{x^2+9}$$

$a)$ For all x, $x^2+9>0$ so there is no vertical asymptote.

$$y=\frac{x-4}{x^2+9}=\frac{\frac{1}{x}-\frac{4}{x^2}}{1+\frac{9}{x^2}}\xrightarrow[x\to\infty]{}0$$

$y=0$ *is asymptote to the curve*

$b)$ $y=k$ intersects the curve so the x-coordinate of the point of intersection

$$\text{satisfies}\,(y=)\,k=\frac{x-4}{x^2+9}$$

$$k(x^2+9)=x-4$$

$$kx^2-x+9k+4=0 \qquad (Eq)$$

$c)$ The equation has real roots when the discriminant is ≥ 0

$$(-1)^2-4\times k\times(9k+4)\geq 0$$

$$1-36k^2-16k\geq 0$$

$$36k^2+16k-1\leq 0$$

$$(18k-1)(2k+1)\leq 0$$

Draw a sketch to support your answer

$$-\frac{1}{2}\leq k\leq\frac{1}{18}$$

$d)$ When $k=-\frac{1}{2}$ *or* $\frac{1}{18}$, the discriminant is 0

and the equation has two equal roots

which means that the line $y=k$ is tangent the curve,

corresponding to a maximum or

minimum value of y (a stationary point)

• *if* $k=-\frac{1}{2}$, (Eq) *becomes* $-\frac{1}{2}x^2-x-\frac{1}{2}=0$

$$x^2+2x+1=0$$

$$(x+1)^2=0 \qquad x=-1$$

One stationary point *is* $(-1,-\frac{1}{2})$

• *if* $k=\frac{1}{18}$, (Eq) *becomes* $\frac{1}{18}x^2-x+\frac{9}{2}=0$

$$x^2-18x+81=0$$

$$(x-9)^2=0 \qquad x=9$$

Another stationary point *is* $(9,\frac{1}{18})$

Part (a) of this question was not always answered as well as expected. Many candidates gave a good explanation for the absence of a vertical asymptote but omitted to attempt the equation of the horizontal asymptote. When both parts were answered, the attempts were usually successful, but an equation *y* = 1 instead of *y* = 0 was quite common.

Part (b) was absolutely straightforward and afforded two easy marks to almost all the candidates.

Part (c) involved inequalities, and while most candidates were familiar with the need to work on the discriminant at this stage, many lost a mark through not clearly and correctly stating the condition for real roots; another mark was lost when the candidate, having legitimately obtained the two critical values, failed to justify the inequalities in the final answer. Again, a sign error in the manipulation of the discriminant often spoiled the attempt to find the two critical values.

It was good to see many candidates gaining full credit in part (d) even when they had struggled unsuccessfully in part (c). The majority of candidates had clearly practised their techniques in this type of question. Sometimes the finding of the *y*-values required an unwarranted amount of effort, since they were known from the outset, and some candidates lost the final mark because of a failure to give the correct *y*-coordinates.

Question 8:

$a)\ f(x) = x^3 + 2x^2 + x - 100000 = 0\ has\ one\ real\ root.$

If $x_1 = 50$ is an approximation of the root

then $x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$ is a better approx.

$f(x_1) = 50^3 + 2\times 50^2 + 50 - 100000 = 30050$

$f'(x) = 3x^2 + 4x + 1$

$f'(x_1) = 3\times 50^2 + 4\times 50 + 1 = 7701$

$$so \quad x_2 = 50 - \frac{30050}{7701} = 46.1\ to\ 1 D.P$$

$b)\ S_n = \sum_{r=1}^{n} r(3r+1) = \sum_{r=1}^{n} 3r^2 + r = 3\sum_{r=1}^{n} r^2 + \sum_{r=1}^{n} r$

$S_n = 3\times \frac{1}{6} n(n+1)(2n+1) + \frac{1}{2} n(n+1)$

$= \frac{1}{2} n(n+1)\left[2n+1+1\right] = \frac{1}{2} n(n+1)(2n+2) = n(n+1)^2$

$ii)\ N(N+1)^2 > 100000$

$N(N^2 + 2N + 1) > 100000$

$N^3 + 2N^2 + N - 100000 > 0$

$c)$ *According to part a)*, an approximation of the root is 46.1

Let try N=45 then $N^3 + 2N^2 + N - 100000 = -4780$

$N = 46$ then $N^3 + 2N^2 + N - 100000 = 1614$

$N = 46$ is the lowest interger so that $S_N > 100000$

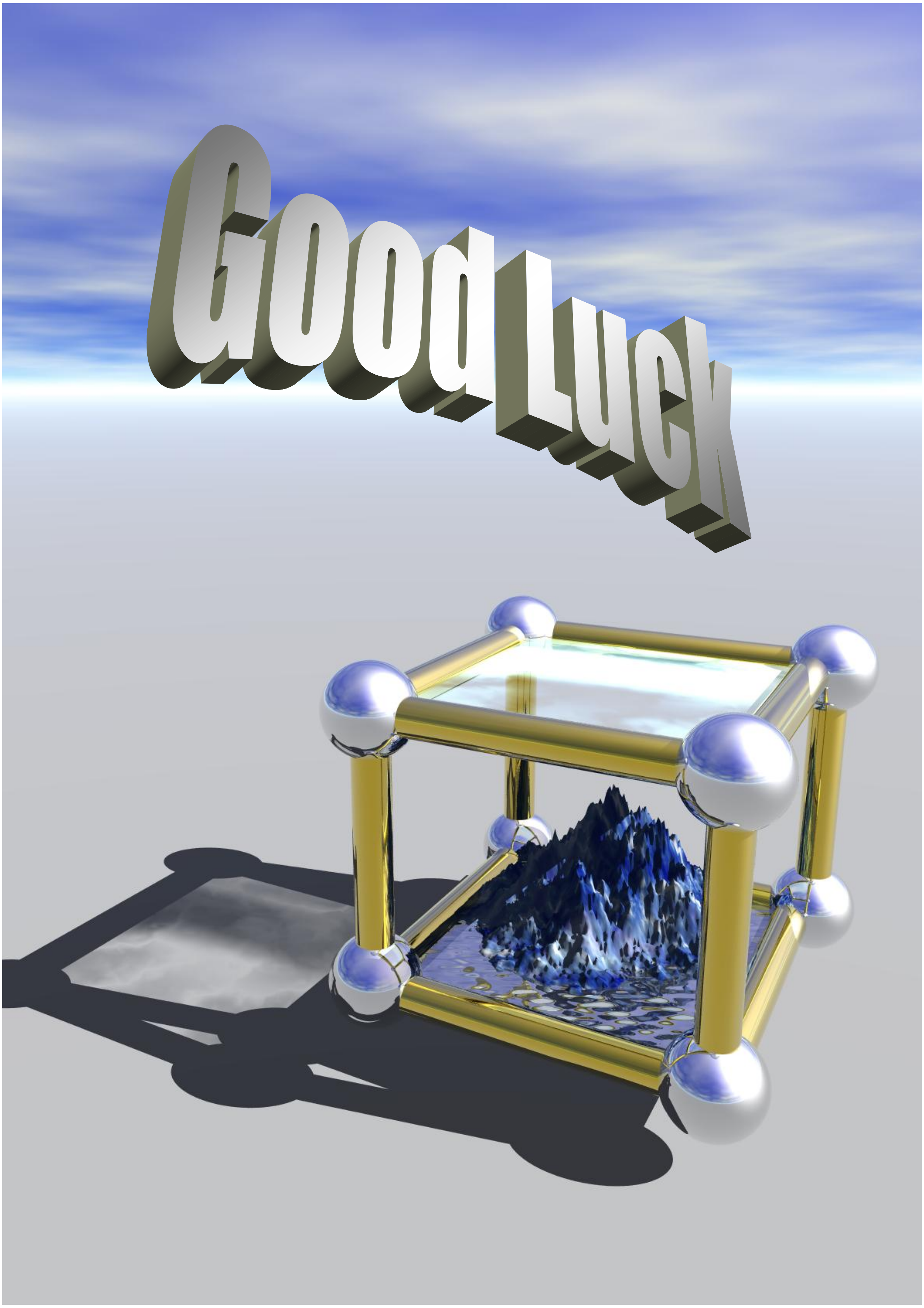
Most candidates applied the Newton–Raphson method correctly in part (a) of this question and obtained a correct value. They then proceeded in many cases to prove the result given in part (b)(i), often by expanding fully before attempting to find the factorised form. Luckily in this case it was not very hard to obtain the factors from the expanded form, particularly as the answer was given. Some candidates, however, lost credit through not showing steps at the end, which they may have thought unnecessary as the answer was 'obvious', but this overlooked the importance of good examination technique when the answer is printed on the question paper. For a similar reason, some candidates lost the one mark available in part (b)(ii), no doubt thinking that the result was 'obvious' and not writing enough steps.

Part (c) required an awareness that *N* must be an integer, and also that the answer to part (a) was not necessarily an accurate guide to the root of the equation given there. Many candidates did a lot of work to find that root more accurately, or indeed quoted the value found on a calculator (45.75), but failed to draw a correct conclusion about the value of *N*. A more successful approach, in general, was to evaluate the left-hand side of the inequality for *N* = 46 and then for *N* = 45, thus showing that the former value was the lowest integer for which the inequality was satisfied.

Grade boundaries

A-level

Code	Title	Max. Scaled Mark	Scaled Mark Grade Boundaries and A* Conversion Points A*	A	B	C	D	E
MFP1	GCE MATHEMATICS UNIT FP1	75	-	63	56	49	43	37


Good Luck